Estimating the Brazilian Output Gap in an MS-DSGE Approach

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January 2021

Abstract

This paper aims to estimate the output gap for Brazil based on a fully specified DSGE model that incorporates Markov-Switching elements (MS-DSGE), to consider the possibility of regime shifts. We propose four versions of the model, among which consider changes in volatilities and in Taylor's rule parameters. In order to compare our output gap estimate with other approaches, we perform prediction tests, both with the central bank's reaction function and with the free price inflation Phillips curve. Our results in the first test indicates that the HP filter estimate performs better in the short and mid term, but the MS-DSGE estimate presented better results in the long-run. In the second exercise, no output gap series stands out among the approaches considered.

JEL Codes: C11, C32, E58, E52.

Keywords: Output Gap, Markov Switching DSGE, Bayeasian Estimation.

1 Introduction

In order to succeed in the monetary policy conduction, it is necessary to have a good evaluation of its effects, as Mishkin (1995) pointed out. With the inflation target regime adoption by Brazil, in June 1999, the mecanism of using the interest rate as a form of monetary policy transmission became a standard activity for the Brazilian Central Bank (BCB, henceforth). This method is also standard in the literature of the last seventy years and emerged in the basic Keynesian model: a contractionist monetary policy raises the real interest rate, which in turn leads to a raise in the capital cost, decreasing the investment level and a subsequent fall in aggregate demand and output. The importance of interest

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rate as an monetary policy instrument also gained strength with Taylor (1995), but in a microeconomic perspective, like the interest rate effects in the individual's decisions.

But since the monetary policy effectiveness in reaching the inflation target depends, among other factors, on the economy's idle capacity, it is essential to have an output gap estimate in order to evaluate the possible monetary policy effects to be adopted. According to Mishkin (2007), there are two reasons that explain the central role of the output gap to the monetary policy: the first is knowing if the policy adopted by the central bank leads to the full employment level. The second is the inflation process dependence on the output gap estimate, because when the output is above its potential, the prices level tends to rise, in response to an excessively high demand, which forces the business and labor market work beyond their maximum efficiency level, to meet the demand level. Alternatively, a negative output gap indicates a lack of demand for goods and services in the economy, so inflation tends to fall. Therefore, output gap estimates are necessary not only to know if the predicted output path by the monetary policy will lead inflation to a stable level, but also if the current monetary policy is efficient.

Currently, Brazil lives the slowest recovery in its history. Over almost four decades (1980-2019), Brazil has faced nine periods of declining GDP[1], but none is as unique as the current one. Of these, the longest and most profound were three: (i) between the first quarter of 1981 and the first quarter of 1983 (1981Q1-1983Q1), in which the economy accumulated a fall of 8,5%; (ii) between 1989Q3-1992Q1, when the GDP shrank 7,7%; and (iii) between 2014Q2-2016Q4, the most recent one, lasting eleven quarters and an accumulated contraction, from peak to valley, of 8,0%. A useful exercise done by Borça, Barboza e Furtado (2019) is to compare the last recession with the average of the previous nine and the other two more intense. If we consider the average of all recessions, we find a relatively small fall (2,5%) and consistent recovery starting four quarters after the beginning. Also, after seven quarters, the economy was already at the same pre-crisis level. In the 1981-1983 crisis, the economy took sixteen quarters to return at your pre-recession level, with a few bumps in the way[2]. With an irregular recovery path, it took eighteen quarters in the 1989-1992 crisis to economy return the 1989Q2 level. But in the 2014-2016 crisis, as Borça, Barboza e Furtado (2019) shows, the contraction was different. First, the decline in activity was continuous for eleven quarters. Second, and unlike the previous ones, after twenty two quarters of the recession beginning, the brazilian economy was still more than 3% below its 2014 level. In other words, compared the average of Brazilian recessions, the 2019Q4 level was 11,2% below the historical pattern of recovery, almost five years later.

"Why has this recovery been unusually slow?" Pires, Borges e Borça Jr (2019), in an attempt to answer this challenging question, work with the possibility of it residing in the great negative output gap caused by the crisis[3]. And one way to investigate this possibility is to analyze its relation with the inflation path. Since 2017, the inflation rate has been closer of the lower band than the target itself[4]. This scenario, according to the authors, suggest that the brazilian monetary policy didn't act in a simetric way in the last few years, a fact that was not highlighted because, apparently, inflation below the target seems to be preferable to inflation above the target[5]. However, this is not what a inflation

[1] According to the Economic Cycle Dating Committee (CODACE).
[2] That period is known as the Latin American countries' external debt crisis.
[3] In addition to particular output gap series, the authors also use the series produced by the Institute for Applied Economic Research (IPEA) and the Independent Fiscal Institution (IFI) as a reference.
[4] In 2019, inflation only hit the target due to a protein price shock of the last two months.
[5] For a better understanding of the theme, refer to Ayres et al. (2019).

target regime prescribes, on the contrary, the role of central banks subjects to this regime is maintain inflation oscillating around the target. Hence, when deviations of it, whether they are negative or positive, are systematic and relevant, this suggests that there is a problem with the calibration of monetary policy.

It is important to highlight the facts that resulted in this scenario. In 2007, the government began to invest, through the National Bank for Economic and Social Development (BNDES), in large national companies, in a movement to increase its competitiveness in the global market, as expressed by Ayres et al. (2019). Besides that, the government also launched a major infrastructure program, called the Growth Accelaration Program (PAC). And with the oil company Petrobras, large investments in the exploration of oil in the pre-salt layer were made. However, neither BNDES nor Petrobras was included in the public-sector fiscal statistics back then, so when such investments started to generate fiscal deficits, they did not appear in government statistics. Furthermore, it also started to implement budget maneuvers to artificially improve its primary surplus, to be in line with fiscal policy goals. The maneuvers was popularly baptized as *creative accounting*.

With the occurrence of the global financial crisis, the government started to bet even more on these policies, in an attempt to stop the recession through countercyclical policies. But after the crisis, the deterioration of public accounts accelerated with the fall in commodity prices, which intensified the country's fiscal fragility. In addition, the government started to intervene in state-owned companies (SOEs) to artificially control inflation, maintaining low prices for fuel and electricity that were sold by them, while other prices in the economy were growing. "The main reason for this intervention is that the government did not want to bear the political burden of reporting higher inflation rates, since it was the government itself that pressured the central bank to reduce nominal interest rates in the first place" (AYRES et al., 2019).

Besides that, the government continued to make use of the *creative accounting* to hide its deficits. By instructing public banks to pay social security pensions and by the incomplete reimbursement of the full amount of these payments, the public banks had losses that should, in fact, be counted as government's primary deficits. These fiscal maneuvers led to the fiscal crisis 2014-2016, which Brazil, now, is trying hard to get out of.

In this context, this paper in an attempt to estimate the brazilian output gap considering the possible changes in the parameters of the economy that occurred due to the conduct of macroeconomic policies in the period. For this, we use the work developed by Oliveira (2013) and take it a step further. In addition to expanding the sample of that work, we adapted the model to a Markov Switching DSGE framework (MS-DSGE), to estimate the output gap and its policy parameter in different regimes. As DSGE models estimation is based on the hypothesis that parameters are invariant to changes in policies and shocks, that is, the parameters are structural in the sense of Lucas's critique, and, motivated by the hypothesis that the 2014-2016 fiscal crisis may have altered the relationship between monetary policy and the idleness of the economy, portrayed by the output gap, the use of an MS-DSGE model converges with this work proposal, since this modeling allows parameters to variate. Another contribution of this study is to enrich the research agenda that aims to represent the brazilian economy in a MS-DSGE approach, such as Gonçalves, Portugal e Aragón (2016), Paranhos e Portugal (2017) and Teixeira (2019).

We incorporate MS elements in a fully specified New Keynesian DSGE model, presented by Hirose, Naganuma et al. (2007), through four different approaches: (i) shifts in stochastic volatilities only, (ii) shifts in Taylor rule policy parameters only, (iii) shifts in both of them, but in the same Markov chain, and, finally, (iv) shifts in both of them, but with the stochastic volatilities following an independent chain, as well as the Taylor rule policy parameters. When we consider shifts in stochastic volatilities, we subject only those related to technology and preference processes, since the flexible-price equilibrium output depends on productivity and demand shocks only.

The model that best captures the recessive moments experienced by the Brazilian economy in the period analyzed by this work is that which allows regime changes only in the parameters of the monetary policy rule (which we refer to as Model 2). The output gap resulting from this estimation interprets the output gap's behavior in a less volatile way than the other proposed models, with well-demarcated periods of recession when compared with the others. In addition, the Model 2 output gap series has a good correlation with the publicly available series for Brazil, which use the production function approach, and the series derived from the HP filter estimation. Also, the comparison with these series demonstrates the contributory potential of this work to the debate of the output gap level in Brazil. In addition to the analytical comparison, we performed a quantitative comparison of the output gap series, through forecasting tests in the framework of a Central Bank reaction function, to verify which gap is the most adherent to the interest rates practiced, and also through a Phillips Curve, to measure the inflationary pressure of the output gap on free prices. The results from the first exercise are favorable to the MS-DSGE approach for long-term forecasts, while for the short term, the results are more adherent to the series derived from the HP Filter. On the other hand, the exercise of forecasting the free items inflation through a Phillips Curve does not present results in favor of one series over another, looking at the general picture, so that for each horizon considered, one series stands out marginally from the others.

Aside from this introduction, this paper is organized as follows. Section 2 presents a brief literature and method review. Section 3 presents and details the MS-DSGE model structure considered to perform the output gap estimation. Section 4 explains the solution method, the definition of the priors and the estimation strategy. Section 5 presents the estimation results, as well as the output gap series comparison. Section 6 performs prediction tests, in order to better evaluate and compare the series, and section 7 concludes.

2 Literature Review

In the context of estimating the output gap, an inherent problem is the fact that output trend data and the potential output is not directly observable. As there is an extensive variety of possibilities for dividing a series into trend and cyclical components and, beside that, the fact that neither economic theory nor econometrics point to a single definition of trend, what we have is a range of methodologies to find an output gap estimate, as highlighted by Álvarez e Gómez-Loscos (2018). In Mishkin (2007), the author divides the estimation of potential output in three basic frameworks, which we will briefly discuss here: aggregate approaches, production function approaches and Dynamic stochastic general equilibrium approaches (DSGE).

2.1 Aggregate Approaches

Aggregate approaches evaluate relationships involving aggregate variables to derive potential output measures. Oliveira (2013) points out that among these approaches, there are two groups: methods with observable and unobservable components. In the first, the most prominent method is the decomposition of Beveridge and Nelson. Beveridge e Nelson (1981) introduce a general method for decomposing a non-stationary time series into a permanent and non-permanent component, allowing both to be stochastic. The procedure, then, was applied to the problem of measuring and dating business cycles in the american economy in the post-war period. An example of the method is found in the work of Evans (1989), who estimated the potential output and the component of the US real GDP cycle from a bivariate VAR for changes in GDP and unemployment rate. But despite the widespread use, according to Álvarez e Gómez-Loscos (2018), three disadvantages of Beveridge and Nelson's decomposition should be highlighted: the first is that the innovations of the trend and the cyclical component are perfectly correlated, since they are driven by the same shock; in addition, the trend component can contain a lot of noise, since the shock variance of the permanent component can be greater than that of data innovation. Third, alternative model's specifications[6] can lead to different trend and cycle decompositions.

In the second group, that of unobservable components, univariate approaches such as Harvey (1985) and Harvey e Jaeger (1993) stand out, in which the authors break down output between trend, cycle and an irregular component, with all the components not correlated with each other. As a deterministic time trend can be considered restrictive, in this approach there is greater flexibility, since the level and slope parameters are allowed to change with time [7]. An application of this method for the case of Brazil can be seen in Pereira (1986). A disadvantage of this approach is that it assumes that output growth is integrated in order of two, which is out of step with the view of most macroeconomists, who consider that output growth is stationary.

Another method of unobservable components is the Hodrick-Prescott (HP) Filter. This procedure, introduced by Hodrick e Prescott (1997) and broadly used, is based on the hypothesis that the trend is stochastic and varies smoothly over time, and also consists on an algorithm to minimize the sum of the squared deviations from a trend. Its main advantage lies in the method's simplicity and uniform structure, which makes it possible to apply it to different economies. However, the HP filter is not a good metric for more recent periods in the sample, so it can not be considered a good candidate for forecasting monetary policy. One example of such methods can be seen in Araujo, Areosa e Guillén (2004). In this work, the authors propose a semi-structural methodology that combines HP filter and the production function approach[8], besides the use of traditional univariate techniques, like deterministic trend, moving average, Beveridge and Nelson's decomposition and unobserved component models, like the Local Level Model, a cyclical one. In order to compare the estimate generated from the different methodologies, the authors use a forward-looking Phillips Curve and a rolling forecast experiment. The main evidence is that the Beveridge and Nelson's decomposition outperforms all the models at all forecast horizons.

Such methods have the advantage of being simple, but they suffer from at least two disadvantages. The first is that they are supported by a variety of statistical assumptions that economic theory provides little corroboration for. An example is the correlation between permanent and cyclical components or whether a random walk is the best model for the permanent component. The second disadvantage is that these purely statistical

[6] ARIMA models, used by Beveridge e Nelson (1981), with similar short-term properties can have different long-term properties.

[7] More specifically, it is assumed that these parameters follow typical random walks.

[8] To be discussed in the next section.

methods do not provide information on the most important potential output's role in the central bank's view, that is, its relationship to a stable rate of inflation.

So, despite such methodologies lead to very different and practical potential output estimates, all with equally reasonable alternative assumptions, as Mishkin (2007) points out, we agree with the view that, in order to have a good measure, it is necessary to have economic theory as a guide, and not only statistics. And it is from this need that other approaches gain space.

2.2 Production Function Approach

A part of the literature is dedicated to estimating the potential output using the production function approach, which generates the estimate of interest from the production factors. Its great advantage is the focus on the various factors that drive the growth of the potential output, instead of the historical behavior of the output growth. Its disaggregated nature allows more data to be used in the estimation, which can be a differential in the occurrence of structural changes in the economy.

As described in Júnior (2005), since potential output can be seen as a supply-side measure of the economy, the production function approach ends up being an intuitive way in this perspective. In this context, it is normally assumed that the economy can be represented by a Cobb-Douglas function with constant returns of scale. But here comes one of the difficulties of the approach, which relates to find a capital stock series for the construction of the function, since the only data we have are gross fixed capital formation and stock variation[9]. The second challenge of this methodology is to define the potential levels of inputs.

This approach is used by many institutions[10] worldwide and in Brazil, of which we highlight the work of Júnior e Caetano (2013) from the Institute of Applied Economic Research (IPEA, henceforth), and of Orair e Bacciotti (2018) from the Independent Fiscal Institution (IFI, henceforth). The first one uses both the production function approach and the HP filter to find the potential output for Brazil in the period 1992-2010. By the method of production function, the authors point to low Brazilian productivity and savings rates as the main reasons for the country's low growth. Furthermore, they did not find much divergence between the two approaches when both series were measured in the reaction function of the monetary authority. On the other hand, the work developed by the IFI highlights the limitation of the HP filter when estimating the output gap in a more robust way through the production function, through use of more reliable estimates of the intermediate variables, such as the TPF and the capital stock. The authors also make use of the Plausibility Tool, proposed by Hristov, Raciborski e Vandermeulen (2017), for the identification of implausible (or counter-intuitive) results and to provide an alternative to the output gap, when necessary.

Another prominent works of this approach are Areosa et al. (2008), which presents a simplified production function that does require TPF or capital and labor stocks data, since the estimation is made by a model that combines a multivariate version of the HP filter objective function with the Phillips Curve; and Borges (2017), which uses a methodology similar to the IFI, and also employed by the European Commission. Also,

[9] Morandi e Reis (2003) estimate the fixed capital stock for Brazil in the period of 1950 to 2001.

[10] Like the IMF (Masi (1997)), the OECD (Giorno et al. (1995)), the European Comission (ECFIN (2006)), the BCE (Willman (2002)), the Congressional Budget Office of USA (Office e Congress (2001)) and also the BCB (Brasil (1999)).

Júnior (2005) provides good survey on the method, in addition to perform an empirical study for Brazil using the production function approach for the period 1980-2000.

2.3 DSGE Approach

To better understand the approach that will be used, it is important to define its context. Christiano, Eichenbaum e Trabandt (2018) claims that the effect of any change in macroeconomic policy is the result of forces operating in different parts of the economy. Thus, the challenge for policymakers is to find the best way to assess the effect of these forces. Among the range of tools that can be chosen to perform such an analysis are the models known as *Dynamic Stochastic General Equilibrium* (DSGE). In practice, the term is used to refer to quantitative models of growth or fluctuations in the business cycle. A classic example is the Real Business Cycle (RBC) model presented by Kydland e Prescott (1982), in which the economy is formed by a perfectly competitive market, where utility maximizing agents are subject to budget and technology constraints. Also, according to Romer (2012), what these models try to achieve is the construction of a microfundamented general equilibrium model and a specification of the shocks that characterize the main macroeconomic fluctuations. In this case, they adopt the idea that such fluctuations are an efficient response of the economy to exogenous technological shocks, so that there would be no need for any form of government intervention.

At the same time, the RBC models did not answer several questions related to macroeconomic policies and of vital importance to policy makers, such as the consequences of different monetary policy rules for the economy as a whole, the effects of alternative exchange rate regimes or the necessary regulations to the financial sector. Thus, DSGE models are built " upon the chassis " of RBC models, as well highlighted by Christiano, Eichenbaum e Trabandt (2018), incorporating nominal frictions in the goods and labor markets. Described as New-Keynesian DSGE models, they express the fact that monetary policy has virtually no impact on real variables in the long run. However, and differently from the RBC models, due to rigid prices and wages, monetary policy is no longer neutral in the short term.

And it is under this context that such models provide a different, although complementary, definition of the potential output, compared to other approaches. In particular, according to Woodford (2011), the potential output can be defined as the level of output that an economy would achieve if the inefficiencies caused by price and wage rigidities were removed, that is, the current production level if wages and prices were completely flexible.

But this perspective of the DSGE approach also has important differences from previous approaches. The works of Neiss e Nelson (2005) and Edge, Kiley e Laforte (2008) show that the properties of the potential output and fluctuations in the output gap may differ from conventional approaches. For example, as is common with most DSGE models, the potential output may fluctuate throughout the business cycle, as this is considered an efficient response to shocks in the economy. In addition, fluctuations in the output gap may be caused by shocks in fiscal policy, changes in household preferences regarding savings and consumption, changes in preferences regarding leisure that affect the labor supply and shocks in terms of trade. But previous approaches, mainly production function, generally assume that such shocks have no important effect on potential output during the business cycle, so that their respective estimates fluctuate less than those extracted from DSGE models.

Another example of this approach is present in the article by Justiniano e Primiceri

(2008), in which the authors use the model to extract both the potential output, defined by them as the level of output under perfect competition, and the natural output, the level of output on price and wage flexibility. They find a smooth potential output, resulting in an output gap very similar to traditional measures, which contradicts the conclusion of Mishkin (2007), Edge, Kiley e Laforte (2008) and Neiss e Nelson (2005). The results for the natural output, on the other hand, show high volatility, due, according to the authors, to the high variability of markup shocks. As the results of Justiniano e Primiceri (2008) point out, the work of Hirose, Naganuma et al. (2007) also obtains an output gap estimate very close to the estimate extracted via the Hodrick-Prescott Filter.

In Fueki et al. (2016), the authors define the potential output as a component of the output level with flexible prices generated only in the event of persistent growth rate shocks. In this case, this efficient long-term output is highly smooth and very similar to conventional measures of potential output, while the efficient short-term output, which would be observed in the absence of nominal rigidity and shocks in price markups and wages, is more volatile, precisely because it behaves similarly to the current output. The effectiveness of the relationship between conventional measures and that based on the model of the work in question is due to the fact that the model incorporates the view of policy makers that an efficient level of output is driven mainly by permanent technological changes.

Despite the excitement about researching the potential output via DSGE models, Mishkin (2007) details its possible disadvantages: DSGE estimates are more dependent on the model than conventional measures, since those depend on the estimated parameters of the model and the estimates of structural shocks that hit the economy [11]. Another disadvantage is that as such models normally assume strong hypotheses to identify shocks in the potential output, the result that these models imply a smaller and less persistent gap than traditional approaches may reflect the idea that other inefficiencies besides price rigidities, like real wages rigidity, are important fluctuations for the output.

Among the advantages of using DSGE models to find an estimate of the output gap, the main one is the deeper structural interpretation that this approach allows, which is essential in the perspective of welfare sought by the policy maker. According to Álvarez e Gómez-Loscos (2018), the joint estimation of potential output and structural shocks in a general equilibrium model allows for a quantitative assessment of inflationary pressures and a more normative analysis of alternative monetary policy measures.

2.3.1 Markov-Switching Models

The Markov-Switching model derived from the work of Hamilton (1989) separates business cycles into two regimes, one of negative growth of the output trend and the other of positive growth, with the economy "back and forth" according to a first-order Markov process. Hamilton's proposal defines the output as the sum of two independent unobservable components, one following a random walk with drift, which evolves according to a two-state Markov process, and the other following an autoregressive process with a unit root. Specifically, the output series is described as:

$$y_t = \tau_t + \nu_t$$

[11] This is clear from the divergent results found in Neiss e Nelson (2005) and Edge, Kiley e Laforte (2008).

in which

$$\begin{aligned} \tau_t &= \tau_{t-1} + \alpha_0 + \alpha_1 S_t \\ prob(S_t = 1 | S_{t-1} = 1) &= p \\ prob(S_t = 0 | S_{t-1} = 1) &= 1 - p \\ prob(S_t = 1 | S_{t-1} = 0) &= q \\ prob(S_t = 0 | S_{t-1} = 0) &= 1 - q \end{aligned}$$

where $S_t = \{0, 1\}$, $S_t = 0$ being an expansion state and $S_t = 1$, a recession. The second component follows an ARIMA $(p, 1, 0)$ process.

As highlighted by Herbst e Schorfheide (2015), Markov-Switching processes can also be incorporated into the DSGE models, forming the MS-DSGE models. In their most practical use, such processes can replace the technological growth rate with a two-state Markov process. The non-linearity of Markov-Switching, still according to Herbst e Schorfheide (2015), can also be added in parameters that are not related to exogenous processes, such as the coefficients of monetary policy. Such modification of the DSGE models would correspond to the same characteristics of the output growth that Hamilton (1989) was able to trace.

One prominent work on the MS-DSGE approach is Liu e Mumtaz (2011), which portray an small open economy for United Kingdom (UK), with agents aware of the possibility of regime switching, in a way that this is considered by them in the time of forming their expectations. The authors consider five versions of the model: (i) no regime switching; (ii) two-state Markov switching in the volatility of the structural shocks; (iii) in addition to the previous one, it its allowed for the parameters of the domestic price inflation Phillips curve to follow an independent two-state Markov process; (iv) regime switching in the import price inflation Phillips curve; (v) regime switching in the open economy Taylor rule; and, finally, (vi) two regimes for all structural parameters in the model, but assumes that agents do not form expectations about the possibility of regime switching. All models that incorporated regime change were preferable to the model with fixed parameters.

In the same way, Chen e MacDonald (2012) also estimated an MS-DSGE model with different versions for the UK, but they went deeper. With the model which performed best, the authors used it to find an optimal monetary rule for the periods analysed, between 1975 and 2010. Their objective was assess how effective were the monetary policy decisions for the economic dynamics. The results point out that the effective monetary policy contributed, at least in part, for the Great Moderation period in UK. The authors also find moments of non-optimal monetary stance in the period.

Gonçalves, Portugal e Aragón (2016) makes use of the work in Liu e Mumtaz (2011) to implement an similar approach for the Brazil case, during the period from 1996 until 2012. The authors assess if the adoption of regime switching parameters would represent better the brazilian economic dynamics. In order to perform that, the authors consider three instead of five versions of regime switching open-economy DSGE model: (i) regime switching in the volatility of exogenous shocks; (ii) in addition to the latter, regime switching in the parameters of the domestic price inflation Phillips curve; and (iii) regime shifts in the volatility of the exogenous shocks and in the parameters of the open economy Taylor rule. In the same way as the original work, Gonçalves, Portugal e Aragón (2016) show that the Markov switching versions were superiors than the one with constant parameters.

Following the international literature, Paranhos e Portugal (2017) is based on the model of Chen e MacDonald (2012). The authors considers regime changes in four different versions: (i) regime switching in Taylor rule parameters only; (ii) shifts in the price stickiness parameter only; (iii) regime changes in stochastic volatilities only; and (iv) a two independent Markov switching process with one specification allowing shifts in the Taylor rule and price stickiness and another one with shifts in stochastic volatilities. But, in an opposite way, the best performing model was the one with no regime changes, which was used as benchmark. However, the authors were able to identify a clear change in the monetary policy stance in 2003, moving from low inflation targeting regime to a high one. This leads them to not reject the hypothesis of regime changes during the analysed period, 2000 until 2016Q3, even though the model comparison results indicate that regime changes were not supported by the data.

The most recent MS-DSGE work for Brazil is found on Teixeira (2019). The author departs from one of the models presented in Galí (2015) textbook, with a fiscal block, and add a Markov-Switching structure to incorporate the possibility of the economy goes from monetary to fiscal dominance, and vice-versa. The model is calibrated, considering DSGE models for the brazilian economy, so only the regime change probabilities are estimated. The work tries to explore the implications of the Fiscal Theory of The Price Level when the economy in under monetary dominance, but households and firms believe there is a chance of switching to fiscal dominance. With this proposal, the author finds that there is a positive probability of 5% of switching to fiscal dominance in Brazil, using data from 2004 until 2018. Also, when this probability is taken into account, there is a significant change in the shocks dynamics, with the monetary policy becoming weaker.

Motivated by the works discussed above, we incorporate Markov-Switching elements in a DSGE model in the next the section, in an attempt to obtain a better estimate for the output gap. The multiple versions of the model described there are of interest because they can better represent the structural changes that occurred in the Brazilian economy in the analyzed period, such as changes in the conduct of monetary policy, especially in transition periods of the presidency at the BCB, as well as a period of political uncertainty, such as the election of ex-president Lula or the impeachment process of ex-president Dilma. Presumably, parameters like the monetary policy rule or the volatility shocks on the Brazilian economy were not constant over the period considered. So, the adoption of the MS-DSGE methodology help us investigate whether and how these structural changes impacted our potential output and, consequently, the path of the output gap.

3 The Markov-Switching DSGE Model

In this section, we present the model used and how we intend to add the Markov-Switching structure in it. The model is the same as Hirose, Naganuma et al. (2007) and Oliveira (2013).

3.1 The Model

The Representative Household

The representative household, who live infinitely and have a multiplicative consumption habit, maximize the following expected utility function:

$$E_t \sum_{i=0}^{\infty} \beta^i D_{t+i} \left[\frac{1}{1-\tau} \left(\frac{C_{t+i}}{C^h_{t+i-1}} \right)^{1-\tau} + \frac{\mu}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N^{1+\eta}_{t+i}}{1+\eta} \right]$$

where D_t is a preference shock that is interpreted as a real demand or IS shock, C_t is the consumption good, C^h_{t+i-1} represents a habit stock consumption with the habit persistence parameter given by $0 < h < 1$, $\frac{M_t}{P_t}$ are the real money balances, $1 - N_t$ is leisure, $0 < \beta < 1$ is the discount factor, τ^{-1} is the intertemporal substitution elasticity, $b > 0$ and $\eta > 0$ are associated with the substitution elasticities with respect to consumption, and $\mu > 0$ and $\chi > 0$ are scale factors.

Given the aggregate price index, the budget constraint is:

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t} \right) N_t + \frac{M_{t-1}}{P_t} + R_{t-1} \left(\frac{B_{t-1}}{P_t} \right) + \Pi_t,$$

where B_t is nominal government bonds that pay the nominal interest rate R_t, $\frac{W_t}{P_t}$ is the real wage, and Π_t is the real profits received from firms, since the household owns these.

The first-order conditions for the household's optimization problem are:

$$\frac{U^*_{C,t}}{C_t} = \beta R_t E_t \left(\frac{U^*_{C,t+1}}{C_{t+1}} \frac{P_t}{P_{t+1}} \right) \tag{1}$$

$$\frac{D_t \mu (\frac{M_t}{P_t})^{-b}}{\frac{U^*_{C,t}}{C_t}} = \frac{R_t - 1}{R_t} \tag{2}$$

$$\frac{D_t \chi N_t^{\eta}}{\frac{U^*_{C,t}}{C_t}} = \frac{W_t}{P_t} \tag{3}$$

where

$$U^*_{C,t} = D_t (C_t / C^h_{t-1})^{1-\tau} - \beta h E_t \left[D_{t+1} (C_{t+1}/C_t)^{1-\tau} \right]. \tag{4}$$

A log-linear approximation of equations (1) and (4) around the steady state, together with the equilibrium condition of *market clearing* $C_t = Y_t$ results in the Euler equation:

$$u^*_{c,t} - y_t = E_t u^*_{c,t+1} - E_t y_{t+1} + r_t - E_t \pi_{t+1}, \tag{5}$$

with

$$u^*_{c,t} = \frac{(1-\tau)}{(1-\beta h)} \left[(1+\beta h^2) y_t - h y_{t-1} - \beta h E_t y_{t+1} \right] + \frac{1}{1-\beta h} d_t - \frac{\beta h}{1-\beta h} E_t d_{t+1} \tag{6}$$

where the lower case letters with time subscriptions represent the percentage deviations from their steady state values. In addition, approximating (3), we arrive at:

$$d_t + \eta n_t - u^*_{c,t} + c_t = w_t - p_t. \tag{7}$$

Firms

The final consumption good Y_t is produced from inputs that are considered intermediate goods, $Y_t(j)$, $j \in [0,1]$ produced by firms in monopolistic competition with the following technology:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\lambda_t}{\lambda_t - 1}} dj \right]^{\frac{\lambda_t}{\lambda_t - 1}},$$

where λ_t is the time-varying elasticity of demand for each intermediate asset. The cost minimization problem of the final good sector provides the demand function for each j good:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\lambda_t} Y_t, \tag{8}$$

and the aggregate price index:

$$P_t = \left[\int_0^1 P_t(j)^{1-\lambda_t} dj \right]^{\frac{1}{1-\lambda_t}}. \tag{9}$$

Each firm faces a downward-sloping demand curve as in (8) for its differentiated product $Y_t(j)$. The production function is linear in the labor input $N_t(j)$:

$$Y_t(j) = A_t N_t(j) \tag{10}$$

where A_t is an exogenous productivity disturbance.

Subject to the production function given by (10), the cost minimization problem of each firm is:

$$\min_{N_t} \frac{W_t}{P_t} N_t + \Phi_t(Y_t(j) - A_t N_t(j)),$$

where Φ_t is the firm's real marginal cost. The first-order indicates that:

$$\Phi_t = \frac{W_t / P_t}{A_t}. \tag{11}$$

According to Calvo (1983), it is assumed that firms can change their price in a given period according to probability $1 - \omega$. Each firm chooses the price $P_t(j)$ to maximize the expected discounted profits:

$$E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} \left[\left(\frac{P_t(j)}{P_{t+i}} \right) Y_{t+i}(j) - \Phi_{t+i} Y_{t+i}(j) \right],$$

where $Q_{t,t+i} = \beta^i \frac{U^*_{C,t+i}/C_{t+i}}{U^*_{C,t}/C_t}$ is the stochastic discount factor. Subject to the demand curve (8) with the *market-clearing* condition $C_t = Y_t$, the first order condition for each firm implies that the optimal price P_t^* chosen by all firms adjusting at time t is:

$$\frac{P_t^*}{P_t} = Z_t \frac{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} Y_{t+i} \Phi_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\lambda_t}}{E_t \sum_{i=0}^{\infty} \omega^i Q_{t,t+i} Y_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\lambda_t - 1}} \tag{12}$$

where $Z_t = \frac{\lambda_t}{\lambda_t - 1}$ express the time-varying markup. From (9), the aggregate price is:

$$P_t = \left[\omega P_{t-1}^{1-\lambda_t} + (1-\omega) P_t^{*1-\lambda_t} \right]^{\frac{1}{1-\lambda_t}}.$$

A linear approximation around the steady state of P_t and P_t^* take us to the New Keynesian Phillips Curve (NKPC):

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\beta\omega)(1-\omega)}{\omega}\varphi_t + \frac{1-\omega}{\omega}(z_t - \beta\omega E_t z_{t+1}), \tag{13}$$

where π_t denotes the inflation rate, $\varphi_t = w_t - p_t - a_t$ is the real marginal cost and z_t the time-varying markup, interpreted as a cost-push shock to the firms' price setting. As defined earlier, lower-case letters with time subscripts represent the percentage deviations from their steady-state values.

Flexible-Price Equilibrium and the Output Gap

The model proposed by Hirose and Naganuma (2007) considers that the output gap is defined as the deviation of the current output from its flexible-prices equilibrium output, which would occur in the absence of cost shocks. In addition, an optimal monetary policy, as pointed out by Woodford (2011), reproduces the flexible-prices equilibrium, a scenario that may occur under the assumption that the government seeks to mitigate the effects of monopolistic competition by providing the necessary subsidies. In other words, the concept of output gap that is considered here, it is a good measure of well-being for policy makers.

Disregarding cost-push shocks for a moment, imagine the case where all firms adjust their prices in every period, that is, consider that there is no more price rigidity. Such a flexible pricing scenario is characterized when $\omega = 0$, $P_t^* = P_t$ and $Z_t = \overline{Z}$ in (12). So, the definition of marginal cost in (11) implies that:

$$\frac{W_t}{P_t} = \frac{A_t}{\overline{Z}}.$$

This relationship, together with the first order condition (3), indicates that the flexible-price equilibrium satisfies:

$$\frac{D_t \chi N_t^{\eta}}{U_{C,t}^*/C_t} = \frac{A_t}{\overline{Z}}.$$

A log-linear approximation around the steady state yields:

$$d_t + \eta n_t^f - u_{c,t}^{*f} + c_t^f = a_t \tag{14}$$

where the superscript f refers to the flexible-price equilibrium. Similarly, the production function (10) can be linearized as:

$$y_t^f = n_t^f + a_t. \tag{15}$$

From (14) and (15), together with the equilibrium condition $y_t^f = c_t^f$, the flexible-price equilibrium output y_t^f can be written as:

$$y_t^f = a_t + \frac{1}{1+\eta}u_{c,t}^{*f} - \frac{1}{1+\eta}d_t, \tag{16}$$

with

$$u_{c,t}^{*f} = \frac{(1-\tau)}{(1-\beta h)}\left[(1+\beta h^2)y_t^f - h y_{t-1}^f - \beta h E_t y_{t+1}^f\right] + \frac{1}{1-\beta h}d_t - \frac{\beta h}{1-\beta h}E_t d_{t+1}. \tag{17}$$

Hence, the flexible-price equilibrium output depends on productivity and demand shocks. Finally, we can now define the output gap as:

$$gap_t = y_t - y_t^f,$$

which measures the percentage deviation of the actual output from the flexible-price equilibrium output.

Monetary Policy

The monetary policy follows a standard Taylor-type rule. As is known, this rule dictates the monetary authority behavior when adjusting the nominal interest rate according to movements in inflation and the output gap of its respective targets. The log-linearized version of the monetary policy rule is:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)\left[\psi_\pi \pi_t + \psi_y (y_t - y_t^f)\right] + \varepsilon_{r,t}, \tag{18}$$

where $\varepsilon_{r,t} \sim N(0, \sigma_r^2)$ captures unanticipated deviations and $0 \leq \rho_r < 1$ determines the degree of interest rate smoothing. $\psi_\pi > 0$, $\psi_y > 0$ and $\varepsilon_{r,t}$ is an exogenous monetary shock.

Exogenous Shock Processes and Equilibrium System

We assume that the demand shock d_t, the cost shock z_t and the productivity shock a_t follow a stationary AR (1) process, as the source of the equilibrium dynamics:

$$d_t = \rho_d d_{t-1} + \varepsilon_{d,t}, \tag{19}$$
$$z_t = \rho_z z_{t-1} + \varepsilon_{z,t}, \tag{20}$$
$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}, \tag{21}$$

wherein $0 \leq \rho_d, \rho_z, \rho_a < 1$ and $\varepsilon_{d,t} \sim N(0, \sigma_d^2)$. Similarly, $\varepsilon_{z,t} \sim N(0, \sigma_z^2)$ and $\varepsilon_{a,t} \sim N(0, \sigma_a^2)$.

3.2 Regime-Switching Exogenous Process

We will work with two possible regimes, each associated with a determined behavior of our regime-dependent parameters. In our proposal, the economy can move between regimes according to an exogenous stochastic first-order Markov-Chain. Consider that $p_{ij} = Prob(s_{t+1} = i | s_t = j)$ where $i, j \in E, R$ and E stands for Expansion while R stands for Recession. In other words and in a simple scenario, when the output gap is positive and when is negative. So, the exogenous Markov-Chain's transition matrix can be defined as:

$$P = \begin{bmatrix} \pi_{EE} & 1-\pi_{EE} \\ 1-\pi_{RR} & \pi_{RR} \end{bmatrix} \tag{22}$$

Regime-dependent parameters are considered in the model by subjecting them to regime change according to the Markov-Chain process described above, with two possible states. In particular, we evaluate four versions of the model described above that allow for (i) regime shifts in the volatility of the exogenous shocks that impact the output gap, namely, the demand and productivity shocks (σ_d and σ_a), (ii) regime shifts in the Taylor rule parameters (ψ_π and ψ_y), (iii) regime shifts in both volatilities and monetary policy parameters ($\sigma_d, \sigma_a, \psi_\pi, \psi_y$), all following the same Markov-Chain, and finally, (iv) the same as the previous one, but with the volatility of the exogenous shocks following one chain and the monetary policy rule parameters following a different one (*independent chains*). This regime-switching structure brings the idea that economic agents know that such transitions can occur and take this in to account when making their decisions.

We are aware that adopting an exogenous Markov process is a limitation of the work, since the ideal would be to consider that the monetary authority can optimally choose which rule to follow according to the current state of the economy, as the proposal in Paranhos e Portugal (2017). However, we believe that the methodology adopted here can contribute to the estimation of the output gap in the Brazilian literature, especially if one wishes to analyze any asymmetry in monetary policy or the main shocks that this unobservable variable is subject to. In addition, this proposal contributes to the MS-DSGE literature for the Brazilian case.

Since the introduction of a Markov process has implications for the solution and estimation, the next section is dedicated to explain the method adopted in this article.

4 Solution and Estimation

4.1 Solution Method

Usually, in the literature, DSGE models are solved by the perturbation techniques developed by Sims (2002) and computationally implemented by *Dynare*[12]. Since our model counts with the presence of non-linearities like switching parameters, Sim's solution method are not useful for us. The non-linearity brought about by the parameters that change over time according to a Markov-Chain has proved to be important to understand changes in monetary policy.

In this sense, large part of the MS-DSGE literature investigates methods for solving these models, providing an accessible "way to study how agents form expectations over possible discrete changes in the economy, such as those in technology and policy" (FOERSTER et al., 2014). As examples, one can resort to the works of Schorfheide (2005), Liu, Waggoner e Zha (2011), Bianchi (2013) and Bianchi e Ilut (2017).

In the approach presented by Farmer, Waggoner e Zha (2011), the authors develop an alternative method to find Minimal State Variable (MSV) equilibria for Markov-Switching linear rational expectations (MSLRE) models, as an algorithm for computing these equilibria. The approach starts with a system of standard linear rational expectations equations that have been obtained by linearizing equilibrium conditions, treating the parameters as constant over time. Next, discrete Markov processes are added to some parameters. As pointed out by Foerster et al. (2014), the resulting MSLRE model may not be compatible with the optimizing behaviour of the economic agents. Also, this approach does not consider higher-order approximations, just because it is built under linear rational expectations models.

Another approach, which is adopted by this work, consider switching parameters in a perturbation method situation, allowing for higher-order approximations and, thus, improving the accuracy of the solution. Examples in this field include Foerster et al. (2014) and Maih (2015). As the latter also provides a *Matlab* toolbox called RISE[13] to adopt the solution method presented in the article, for convenience, we follow its methodology.

[12] See Barillas et al. (2010)
[13] RISE is a *Matlab*-based object-oriented toolbox. It is available, free of charge, in this link

A general MS-DSGE problem is as follows:

$$E_t \sum_{s_{t+1}=1}^{h} \pi_{s_t,s_{t+1}}(\mathcal{I}_t)\tilde{d}_{s_t}(v) = 0 \quad \forall s_t \in \{E, R\} \tag{23}$$

where E_t is the expectation operator conditional on information available up to period t, $\tilde{d}_{s_t} : \mathbb{R}^{n_v} \longrightarrow \mathbb{R}^{n_d}$, is a $n_d \times 1$ vector of possibly nonlinear functions of their argument v, $s_t = 1, 2, ..., h$ is the regime in period t and $\pi_{s_t,s_{t+1}}$ is the probability of transitioning from regime s_t to s_{t+1} in the next period. This probability can depend on the information set $\mathcal{I}_t$ at time t, in other words, it can be endogenous. But, in this work, we assume that the probabilities are completely exogenous and do not depend on $\mathcal{I}_t$.

In its turn, the $n_v \times 1$ vector is defined as

$$v \equiv [b_{t+1}(s_{t+1})' \quad f_{t+1}(s_{t+1})' \quad e_t(s_t)' \quad p_t(s_t)' \quad b_t(s_t)' \quad f_t(s_t)' \quad p'_{t-1} \quad b'_{t-1} \quad \varepsilon'_t \quad \theta'_{s_{t+1}}]'$$

where:

- e_t is a $n_e \times 1$ vector of estative variables, which appear in the model at time t only.
- f_t is a $n_f \times 1$ vector of forward-looking variables, appearing in the model both at time t and at time $t + 1$.
- p_t is a $n_p \times 1$ vector of predetermined variables, which appear in the model at time t and $t - 1$.
- b_t is a $n_b \times 1$ vector of "both" variables. That means those variables that are both predetermined and forward-looking.
- ε_t is a $n_\varepsilon \times 1$ vector of shocks with $\varepsilon_t \sim N(0, I_{n_\varepsilon})$.
- $\theta_{s_{t+1}}$ is a $n_\theta \times 1$ vector of switching parameters appearing in a forward looking way in the model.

Assuming that the agents have information for all or some of the shocks $k \geq 0$ periods ahead and also adding a perturbation parameter σ, we define an $n_z \times 1$ vector of state variables as

$$z_t \equiv [p'_{t-1} \quad b'_{t-1} \quad \sigma \quad \varepsilon'_t \quad \varepsilon'_{t+1} \quad \dots \quad \varepsilon'_{t+k}]' \tag{24}$$

of dimension $n_z = n_p + n_b + (k + 1)n_\varepsilon + 1$.

The problem described in (23) can be solved by a set of policy and transition functions, dependent on each regime. Denoting by $y_t(s_t)$ the $n_y \times 1$ vector of all the endogenous variables, where $n_y = n_e + n_p + n_b + n_f$, we want to find solutions of the type:

$$y_t(s_t) = \begin{bmatrix} e_t(s_t) \\ p_t(s_t) \\ b_t(s_t) \\ f_t(s_t) \end{bmatrix} = \mathcal{T}^{s_t}(z_t) \equiv \begin{bmatrix} \mathcal{E}^{s_t}(z_t) \\ \mathcal{P}^{s_t}(z_t) \\ \mathcal{B}^{s_t}(z_t) \\ \mathcal{F}^{s_t}(z_t) \end{bmatrix} \quad \forall s_t \in \{E, R\}. \tag{25}$$

$\mathcal{T}^{s_t}(z_t)$ is a vector containing all policy and transition functions. Since a closed form solution to (23) is usually not available, we use the perturbations methods derived by Maih (2015) to find a Taylor approximation of the policy vector $\mathcal{T}^{s_t}(z_t)$.

A first-order approximation to the solution takes the following form:

$$\mathcal{T}^{r_t}(z) \simeq \mathcal{T}^{r_t}(\bar{z}_{r_t}) + \mathcal{T}_z^{r_t}(z_t - \bar{z}_{r_t}) \tag{26}$$

and a second-order one is:

$$\mathcal{T}^{r_t}(z) \simeq \mathcal{T}^{r_t}(\bar{z}_{r_t}) + \mathcal{T}_z^{r_t}(z_t - \bar{z}_{r_t}) + \frac{1}{2}\mathcal{T}_{zz}^{r_t}(z_t - \bar{z}_{r_t})^{\otimes 2} \tag{27}$$

where $\mathcal{T}_z^{r_t}(\cdot)$ is a vector of gradients and $\mathcal{T}_{zz}^{r_t}(\cdot)$ is a vector stacking Hessian matrices. We use the notation $A^{\otimes k}$ as a shorthand for $A \otimes A \otimes \ldots \otimes A$, k times, and $\bar{z}_r$ is the point around which the approximation is done[14].

In the perturbation technique, the first step is to choose the approximation point. In a constant-parameter scenario, like in the DSGE models, the approximation is typically done around the steady state, because it is the point to which the system will converge in the absence of future shocks. But in a switching-regime context, like in the MS-DSGE models, the choice is not so clear. Foerster et al. (2014) indicate to take a perturbation around its ergodic mean. This, in turn, need not to be an resting point, so Maih (2015) propose two further options: approximate around the regime-specific steady states or around an arbitrary point. As the technique developed by him proposes to follow the first option, this is the path we have adopted. Even if the system is not stable at the mean in a certain regime, at least this approach assumes that if the system happens to be exactly at one its regime-specifics means and in the absence of any further shocks, the system will stay in this point. We compute those means by solving:

$$\tilde{d}_{r_t}(b_t(r_t), f_t(r_t), s_t(r_t), p_t(r_t), b_t(r_t), f_t(r_t), p_t(r_t), b_t(r_t), 0, \theta_{r_t}) = 0 \tag{28}$$

The intuition behind this path is because the relevant issues for rational agents living in a specific state of the system at some point in time is to be protected of the possibility of switching to a different state. Also, the point to which the system returns is important for forecasting. As an example, many inflation targeting countries have moved from a high inflation scenario to a lower one. If the system were approximated around the ergodic mean, in this context this would imply that the unconditional forecasts would be pulled towards a level that is consistently high than the recent past of the inflation, which would probably generate forecast errors. Therefore, the ergodic mean is not necessarily a resting point.

4.2 Estimation

Data

To perform the estimation, quarterly data collected from the BCB and the Brazilian Institute of Geography and Statistics (IBGE) were used. The period ranges from 2000Q1 to 2019Q4. The data consists of the following variables:

- **GDP**: quarterly GDP per capita growth. For the GDP series, the seasonally adjusted series with chained values at 1995 prices was used, avaiable at IBGE. For the labor force, a linked series was created combining data from the economically active population (PEA) obtained through the PME/IBGE until 2011, with the labor force series available by PNADC/IBGE. So, the series is the log difference of the GDP per capita scaled by 100.

[14] On behalf of a non-exhaustive reading, the reader is referred to Maih (2015) for details on how to find $\mathcal{T}_z^{r_t}(\cdot)$ and $\mathcal{T}_{zz}^{r_t}(\cdot)$

- **Inflation**: the inflation rate is given by the log difference of the consumer price index IPCA scaled by 400. The IPCA series is seasonally adjusted using the X13 filter.

- **Interest Rate**: the effective Selic interest rate accumulated in the quarter, avaiable at BCB.[15].

The measurement equation which relates the series presented above with the model is:

$$\begin{bmatrix} \Delta GDP_t \\ INF_t \\ INT_t \end{bmatrix} = \begin{bmatrix} \gamma^* + \Delta y_t \\ \pi^* + 4\pi_t \\ r^* + \pi^* + 4r_t \end{bmatrix} \tag{29}$$

The parameters γ^*, π^* and r^* are the steady state values of output growth, inflation and interest rates, respectively, and they are estimated with the other model parameters, according to the methodology presented below.

Methodology

Following the DSGE estimation literature, the Bayesian approach is used to estimate the proposed model. As is known, the methodology consists in assuming a prior distribution about the parameters to be estimated and then updates them using the likelihood function. This step yields to the posterior distribution, which is simulated by a Markov-Chain Monte Carlo (MCMC) algorithm. We performed 200,000 iterations of the MCMC algorithm with 25% of those being discarded in the burn-in period and with one chain. At last, the posterior distribution is used by the Metropolis-Hasting algorithm for posterior simulation. The acceptance rate was 30%.

But in a switching-parameter case like the MS-DSGE, the calculation of the likelihood function is dependent on the past of the regimes. For this reason, the Kalman filter can not be directly applied, since the number of possible likelihoods grows exponentially with the sample size. So, to find the likelihood function, we rely on Kim's filter (Kim, Nelson et al. (1999)), faithfully presented below.

Define $\mathcal{S}$ as the set of regimes considered. Let Ψ_{t-1} denote the vector of observed variable in time $t-1$ and let β_t be the state vector. In a situation with no regime-switching, the Kalman filter is used to find a forecast of the unobserved β_t based on Ψ_{t-1}, denoted by $\beta_{t|t-1}$. Formally,

$$\beta_{t|t-1} = E[\beta_t|\Psi_{t-1}] \tag{30}$$

Also, the $P_{t|t-1}$ matrix express the mean square error of the forecast:

$$P_{t|t-1} = E[(\beta_t - \beta_{t|t-1})(\beta_t - \beta_{t|t-1})'|\Psi_{t-1}] \tag{31}$$

But in a Markov-Switching context, the aim is to find a possible dynamic of β_t conditional not just on Ψ_{t-1}, but also on the regime s_t being j and on s_{t-1} being i, where $i, j \in \mathcal{S}$:

$$\beta_{t|t-1}^{(i,j)} = E[\beta_t|\Psi_{t-1}, s_t = j, s_{t-1} = i]. \tag{32}$$

[15] Number series 4189

This procedure calculates $|\mathcal{S}|^2$ forecasts as above for each date t, corresponding to every possible value of i and j. In its turn, for each one of these forecasts there are $|\mathcal{S}|^2$ different mean squared error matrices:

$$P_{t|t-1}^{(i,j)} = E[(\beta_t - \beta_{t|t-1})(\beta_t - \beta_{t|t-1})'|\Psi_{t-1}, s_t = j, s_{t-1} = i] \tag{33}$$

Conditional on $s_{t-1} = i$ and $s_t = j$, the Kalman filter procedure pursue the following steps:

1. *Prediction*: Forecast the state and the associated variance:
   - $\beta_{t|t-1}^{(i,j)} = \tilde{\mu}_j + F_j \beta_{t-1|t-1}^i$
   - $P_{t|t-1}^{(i,j)} = F_j P_{t-1|t-1}^i F_j' + G_j Q_j^* G_j'$
2. *Adjustment*: Calculate forecast error and the associated variance:
   - $\eta_{t|t-1}^{(i,j)} = y_t - H_j \beta_{t|t-1}^{(i,j)} - A_j z_t$
   - $f_{t|t-1}^{(i,j)} = H_j P_{t|t-1}^{(i,j)} H_j' + R_j$
3. *Updating*: Update the estimate of the state and the corresponding variance:
   - $\beta_{t|t}^{(i,j)} = \beta_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)} H_j' [f_{t|t-1}^{(i,j)}]^{-1} \eta_{t|t-1}^{(i,j)}$
   - $P_{t|t}^{(i,j)} = (I - P_{t|t-1}^{(i,j)} H_j' [f_{t|t-1}^{(i,j)}]^{-1} H_j) P_{t|t-1}^{(i,j)}$

where $\beta_{t-1|t-1}^i$ is an inference about β_{t-1} conditional on information up to time $t-1$ and $s_{t-1} = i$. $\beta_{t|t-1}^{(i,j)}$ is an inference of β_t based on information up to time $t-1$, conditional on $s_t = j$ and $s_{t-1} = i$. $P_{t|t-1}^{(i,j)}$ is the mean square error matrix of $\beta_{t|t-1}^{(i,j)}$ conditional on $s_t = j$ and $s_{t-1} = i$. $\eta_{t|t-1}^{(i,j)}$ is the conditional forecast error of y_t based on information up to time $t-1$, given $s_{t-1} = i$ and $s_t = j$. At last, $f_{t|t-1}^{(i,j)}$ is the conditional variance of the forecast error $\eta_{t|t-1}^{(i,j)}$.

Because the main problem is that the number of likelihood calculations grow exponentially as the number of observations increase, the Kim's filter proposal is to reduce the $\mathcal{S} \times \mathcal{S}$ posteriors ($\beta_{t|t}^{(i,j)}$ and $P_{t|t}^{(i,j)}$) into $\mathcal{S}$ posteriors ($\beta_{t|t}^j$ and $P_{t|t}^j$) to perform the above steps. So, to make the filter operable, Kim, Nelson et al. (1999) presents the following approximations:

$$\beta_{t|t}^j = \frac{\sum_{i=1}^{\mathcal{S}} Pr[s_{t-1} = i, s_t = j|\Psi_t] \beta_{t|t}^{(i,j)}}{Pr[s_t = j|\Psi_t]} \tag{34}$$

and

$$P_{t|t}^j = \frac{\sum_{i=1}^{\mathcal{S}} Pr[s_{t-1} = i, s_t = j|\Psi_t] \{P_{t|t}^{(i,j)} + (\beta_{t|t}^j - \beta_{t|t}^{(i,j)})(\beta_{t|t}^j - \beta_{t|t}^{(i,j)})'\}}{Pr[s_t = j|\Psi_t]} \tag{35}$$

Equations (34) and (35) are used at the end of each iteration to collapse the $\mathcal{S} \times \mathcal{S}$ posteriors of the updating phase into $\mathcal{S} \times 1$ to make the filter functional.

To complete the Kalman filter - now the Kim's filter -, it is necessary to make inference about the probabilities that show up in equations (34) and (35), what will allow us to obtain filtered and smoothed regime probabilities. The procedure is carried out in 3 stages:

1. At the beginning of the t-th iteration, given the $Pr[s_{t-1}=i|\Psi_{t-1}]$ term ($i=1,2,...,\mathcal{S}$), we are able to calculate

$$Pr[s_t=j,s_{t-1}=i|\Psi_{t-1}] = Pr[s_t=j|s_{t-1}=i] \times Pr[s_{t-1}=i|\Psi_{t-1}] \tag{36}$$

   where $Pr[s_t=j|s_{t-1}]$ is the transition probability on matrix (22).

2. Find the joint density of y_t, s_t and s_{t-1}:

$$f(y_t,s_t=j,s_{t-1}=i|\Psi_{t-1}) = f(y_t|s_t=j,s_{t-1}=i,\Psi_{t-1}) \times Pr[s_t=j,s_{t-1}=i|\Psi_{t-1}] \tag{37}$$

   We use this to obtain the marginal density of y_t:

$$f(y_t|\Psi_{t-1}) = \sum_{j=1}^{\mathcal{S}}\sum_{i=1}^{\mathcal{S}} f(y_t|s_t=j,s_{t-1}=i,\Psi_{t-1}) \times Pr[s_t=j,s_{t-1}=i|\Psi_{t-1}] \tag{38}$$

   where the conditional density $f(y_t|s_t=j,s_{t-1}=i,\Psi_{t-1})$ is obtained based on the prediction error decomposition.

3. Once data on period t is observed, we can update the probability of step 1:

$$Pr[s_{t-1}=i,s_t=j|\Psi_t] = \frac{f(y_t|s_{t-1}=i,s_t=j,\Psi_{t-1})f(s_{t-1}=i,s_t=j|\Psi_{t-1})}{f(y_t|\Psi_{t-1})} \tag{39}$$

   with

$$Pr[s_t=j|\Psi_t] = \sum_{i=1}^{\mathcal{S}} Pr[s_{t-1}=i,s_t=j|\Psi_t] \tag{40}$$

Now we can get the filtered and smoothed probabilities:

Filtered Probability: in the beginning of the t-th iteration, given $Pr[s_t=i|\Psi_{t-1}]$, where $i,j \in \{E,R\}$. We define the filtered probability as:

$$\begin{aligned} Pr[s_t=j|\Psi_{t-1}] &= \sum_{i\in\{E,R\}} Pr[s_t=j,s_{t-1}=i|\Psi_{t-1}] & (41)\\ &= \sum_{i\in\{E,R\}} Pr[s_t=j|s_{t-1}=i]Pr[s_{t-1}=i|\Psi_{t-1}] & (42)\end{aligned}$$

Smoothed Probability: once the parameters are estimated, we can make inferences on s_t and β_t given all the information in the sample. Formally, the smoothed probability is:

$$Pr[s_t=j|\Psi_T] = \sum_{k\in\{E,R\}} Pr[s_t=j,s_{t+1}=k|\Psi_T] \tag{43}$$

The full calculation requires the following derivation of the joint probability of $s_t=j$ and $s_{t+1}=k$ based on all the sample:

$$\begin{aligned} Pr[s_t=j,s_{t+1}=k|\Psi_T] &= Pr[s_{t+1}=k|\Psi_T] \times Pr[s_t=j|s_{t+1}=k,\Psi_T]\\ &\approx Pr[s_{t+1}=k|\Psi_T] \times Pr[s_t=j|s_{t+1}=k,\Psi_t]\\ &= \frac{Pr[s_{t+1}=k|\Psi_T] \times Pr[s_t=j,s_{t+1}=k|\Psi_t]}{Pr[s_{t+1}=k|\Psi_t]}\\ &= \frac{Pr[s_{t+1}=k|\Psi_T] \times Pr[s_t=j|\Psi_T] \times Pr[s_{t+1}=k|s_t=j]}{Pr[s_{t+1}=k|\Psi_t]}\end{aligned}$$

So, just as Kim, Nelson et al. (1999) synthesized, the filter "*may actually be considered a combination of extended versions of the Kalman filter and the Hamilton filter, along with appropriate approximations*". This occurs because if we had to use the Kalman filter, we would need to calculate the likelihood for each regime in every iteration, which will lead us to 2^t likelihoods, since we consider two regimes. This scenario would generate a high computational cost, as the number of observations increases. In its turn, the Hamilton filter contributes with its evaluation of the transition probabilities across all the considered regime paths, in each iteration, and the possibility of using them to build weighted average likelihoods allows us to complete the estimation.

Priors

Table 1 – Priors, means and 90% Credibility Intervals

	Prior				Posterior		
Parameter	Density	Domain	Mean	Std. Dev	Mean	5%	95%
τ	Gamma	$\Re^+$	1.86	0.15	1.8174	1.5814	2.0697
h	Beta	$[0,1)$	0.50	0.10	0.5303	0.3294	0.7276
ω	Beta	$[0,1)$	0.66	0.05	0.7297	0.6755	0.7781
r^*	Gamma	$\Re^+$	4.42	1.00	5.2499	4.1864	6.3286
η	Gamma	$\Re^+$	1.64	0.25	1.5879	1.2735	1.9557
ψ_π (1)	Gamma	$\Re^+$	1.43	0.10	1.5355	1.3511	1.7439
ψ_π (2)	Gamma	$\Re^+$	0.53	0.10	-	-	-
ψ_y (1)	Gamma	$\Re^+$	0.28	0.05	0.3276	0.2237	0.4407
ψ_y (2)	Gamma	$\Re^+$	0.48	0.10	-	-	-
ρ_r	Beta	$[0,1)$	0.80	0.10	0.8571	0.8331	0.8788
ρ_d	Beta	$[0,1)$	0.50	0.15	0.7767	0.6666	0.8609
ρ_z	Beta	$[0,1)$	0.50	0.15	0.5199	0.2745	0.7542
ρ_a	Beta	$[0,1)$	0.50	0.15	0.9153	0.8572	0.9592
γ^*	Normal	$\Re^+$	0.11	0.05	0.0738	0.0103	0.1379
π^*	Gamma	$\Re^+$	6.00	2.00	6.1685	4.8005	7.5014
σ_r	Weibull	$\Re^+$	0.45	0.30	0.2522	0.2168	0.2936
σ_z	Weibull	$\Re^+$	0.45	0.30	0.7100	0.0525	1.9106
σ_d	Weibull	$\Re^+$	0.45	0.30	3.0734	2.4253	3.7675
σ_a	Weibull	$\Re^+$	0.45	0.30	2.3557	1.8733	2.9475
p_{12}	Beta	$[0,1)$	0.10	0.05	-	-	-
p_{21}	Beta	$[0,1)$	0.10	0.05	-	-	-
q_{12}	Beta	$[0,1)$	0.10	0.05	-	-	-
q_{21}	Beta	$[0,1)$	0.10	0.05	-	-	-

The prior distributions was determined based on evidences from national and international literature. Description (1) refers to the prior adopted in regime 1, and similarly, distinction (2) in regime 2. The priors for the Taylor rule parameters are in line with the posterior estimates for those parameters by Gonçalves, Portugal e Aragón (2016). The steady state interest rate prior mean was selected according to Paranhos e Portugal (2017), which in turn defines the discount factor[16], by the relation $\beta = [exp(r^*/400)]^{-1}$. For the parameters γ^* and π^*, the priors were based on historical averages of the actual data. The shocks standard deviations followed the propose presented by Tao Zha in his tutorial RISE codes[17]. We tried to adopt the conventional inverse gamma distribution, but the

[16] The value estimated for β is 0.989, as presented by Castro et al. (2015).
[17] They are avaiable with the RISE toolbox instalation.

results were better with this proposal. For the remaining parameters, the work of Hirose, Naganuma et al. (2007) and Oliveira (2013) were followed, with the exception of the degree of interest rate smoothing (ρ_r), for which the posterior average of the national literature was adopted. We also followed Paranhos e Portugal (2017) for the prior specification of the transition probabilities. Table 1 summarize the priors and presents the estimation results for the scenario with no regime changes, which we call Model 0.

5 Results

The Model 0 estimation results show us estimates in line with the values presented in the national literature. It can be observed that the steady state inflation rate (π^*) and the steady state real interest rate (r^*) were close to there historical averages. These values indicate a nominal interest rate of 11,42%, while our sample presents mean value of 12,34%. Regarding the γ^* parameter, the posterior estimate was below the historical mean, used as the prior, but respected the interval error band. According to the posterior estimates of the Taylor rule parameters, it appears that the BCB adopts an anti-inflationary stance, and it also take in consideration the output gap level, since its response coefficient was bigger that the prior. Also, the interest rate smoothing (ρ_r) estimate was in line with observed in the literature, showing that movements in monetary policy tend to be smooth, as expected. The ω parameter, also known as the Calvo parameter, presented a mean value of 0.73, which indicates that firms change their prices approximately every three and a half quarter. It it worth mentioning that the productivity shock exhibited a persistence (ρ_a) greater than 0.90 and that the demand and productivity volatilities (σ_d and σ_a) were bigger than the others, which may indicate a greater share of their respective shocks.

In its turn, Table 2 summarizes the posterior estimates for the four MS-DSGE versions, which we call Model 1 to Model 4.

Model 1 allows changes only in standard deviations of the productivity and demand exogenous shocks. As shown in the table 2, greater volatilities characterize regime 2 and we point a little overlap across regimes in the confidence intervals. Also, the standard deviations almost double in regime 2. The filtered and smoothed probabilities of this regimes, as the output gap series of Model 1, are presented in the Appendix. The model was able to capture the instability moments of the beginning of the decade, with the election of Lula, but does not capture other similar moments, like the 2008 financial crisis and the economic and political crisis in Brazil during the second term of President Dilma. We believe that this is due to our model representing a closed economy, so that it does not capture exchange rate movements. Besides that, the output gap series does not capture all the recessive periods determined by CODACE[18]. The remaining parameters shows very similarity with Model 0.

The second model, which allows changes only in the Taylor rule parameters, indicates that in regime 2, the monetary authority follows a low inflation targeting regime, with more participation of the output gap level. This can be observed by the difference of the inflation response coefficient between the regimes (from 1.46 to 0.54), as the opposite occurs with the output gap response coefficient (from 0.30 to 0.63). Also, the transition probabilities at the bottom of Table 2 was in line with the prior. The filtered and smoothed probabilities of regime 2, characterized as a low inflation targeting regime, practically does not occur with this model configuration. We expected the model to be able to capture

[18] Economic Cycles Dating Committee/IBRE

Table 2 – Posterior means and 90% Error bands - Markov-Switching models

Parameter	Model 1		Model 2		Model 3		Model 4	
	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2	Regime 1	Regime 2
τ	1.7694	-	1.8264	-	1.7604	-	1.7925	-
	(1.5419, 2.0086)		(1.6073, 2.0724)		(1.5337, 2.0005)		(1.5712, 2.0291)	
h	0.5381	-	0.6350	-	0.6210	-	0.6852	-
	(0.3060, 0.7671)		(0.4427, 0.8024)		(0.4202, 0.7987)		(0.4892, 0.8537)	
ω	0.7229	-	0.7829	-	0.7755	-	0.7873	-
	(0.6525, 0.7846)		(0.7343, 0.8270)		(0.7312, 0.8148)		(0.7397, 0.8330)	
r^*	4.9420	-	5.1883	-	4.9411	-	4.7547	-
	(3.7912, 6.1460)		(4.0156, 6.3974)		(3.7857, 6.1240)		(3.6121, 5.9287)	
η	1.5791	-	1.5474	-	1.5432	-	1.5352	-
	(1.2531, 1.9312)		(1.2232, 1.9086)		(1.2272, 1.8934)		(1.2185, 1.8841)	
ψ_π	1.5597	-	1.4630	0.5449	1.4968	0.5710	1.4631	0.5310
	(1.3600, 1.7654)		(1.2629, 1.6709)	(0.3810, 0.7274)	(1.2919, 1.7133)	(0.4087, 0.7803)	(1.2589, 1.6769)	(0.3643, 0.7141)
ψ_y	0.3225	-	0.2994	0.6333	0.3210	0.5807	0.3016	0.6133
	(0.2198, 0.4434)		(0.1946, 0.4218)	(0.4294, 0.8589)	(0.2189, 0.4382)	(0.4145, 0.7711)	(0.2016, 0.4134)	(0.4193, 0.8351)
ρ_r	0.8564	-	0.8535	-	0.8559	-	0.8565	-
	(0.8306, 0.8792)		(0.8292, 0.8756)		(0.8330, 0.8765)		(0.8314, 0.8786)	
ρ_d	0.7852	-	0.7498	-	0.7583	-	0.7276	-
	(0.6447, 0.8843)		(0.6343, 0.8414)		(0.6363, 0.8500)		(0.5756, 0.8360)	
ρ_z	0.5013	-	0.4563	-	0.4825	-	0.4660	-
	(0.2522, 0.7501)		(0.2416, 0.6759)		(0.2434, 0.7343)		(0.2371, 0.7126)	
ρ_a	0.9270	-	0.8586	-	0.8816	-	0.8598	-
	(0.8569, 0.9737)		(0.7944, 0.9226)		(0.8160, 0.9448)		(0.7937, 0.9231)	
γ_*	0.0694	-	0.0973	-	0.0836	-	0.0842	-
	(0.0034, 0.1345)		(0.0372, 0.1576)		(0.0223, 0.1459)		(0.0240, 0.1437)	
π_*	5.8611	-	6.2870	-	5.7993	-	5.8519	-
	(4.4353, 7.2618)		(5.1303, 7.4568)		(4.6561, 6.9779)		(4.7385, 6.9136)	
σ_r	0.2557	-	0.2154	-	0.2431	-	0.2140	-
	(0.2187, 0.2906)		(0.1506, 0.2689)		(0.1904, 0.2891)		(0.1351, 0.2675)	
σ_z	0.5226	-	1.4540	-	0.8063	-	1.2842	-
	(0.0444, 1.4850)		(0.0972, 3.2114)		(0.0564, 2.3383)		(0.0863, 3.1802)	
σ_a	1.8779	3.6543	3.3823	-	1.9977	4.8032	2.0652	5.1275
	(1.0662, 2.5570)	(2.1061, 6.0236)	(2.5633, 4.2474)	-	(0.9885, 2.7382)	(2.7627, 7.6830)	(1.0418, 2.8985)	(2.9586, 8.0221)
σ_d	2.5689	4.7805	2.8353	-	2.7006	4.5717	2.8102	4.3969
	(1.5546, 4.0028)	(2.3506, 8.4031)	(2.0718, 3.7777)	-	(1.8990, 3.5124)	(2.9336, 6.9618)	(1.9506, 3.7234)	(2.6516, 7.0295)
p_{12}	-	0.0727	-	0.0969	-	0.0708	-	0.0712
		(0.0233, 0.1534)		(0.0303, 0.1978)		(0.0245, 0.1446)		(0.0250, 0.1476)
p_{21}	0.1124	-	0.0893	-	0.1195	-	0.1036	-
	(0.0340, 0.2120)		(0.0231, 0.1953)		(0.0366, 0.2181)		(0.0339, 0.2015)	
q_{12}	-	-	-	-	-	-	-	0.1013
								(0.0274, 0.1897)
q_{21}	-	-	-	-	-	-	0.0945	-
							(0.0239, 0.1929)	

moments of low inflation targeting, like Paranhos e Portugal (2017) does. It is valid to point that with other priors definitions, these moments appears, but the output gap series, in its turn, was not able to portray the economic cycles, especially the recessive periods. This happened not only with this version model, but with the others too. In this sense, we prioritize the results that presented the best output gap series, even if the probabilities did not capture all the expected moments according to the structure of the model.

In Model 3, in which both volatilities and Taylor rule parameters can switch, but following the same Markov-Chain, regime 2 is characterized by greater volatility and low pursuit of the inflation target: σ_a is more than double and σ_d is quite double the values of Regime 1; also, the monetary policy parameters behave similarly to Model 2, with the inflation response coefficient going from 1.50 to 0.57, and the output gap response coefficient rising from 0.32 to 0.58. This behavior is also in line with the results found by Gonçalves, Portugal e Aragón (2016) and Paranhos e Portugal (2017). The filtered and smoothed probabilities are able to show some moments of instability, as the uncertainty regarding the macroeconomic policy of the early 2000s and the instability of the second government Dilma, but the output gap series does not carry the recession moments of the analyzed period.

Model 4, at last, permits regime changes in both volatilities and monetary policy parameters, but combining two independent Markov-Chains. The posterior results are very similar with the other models, in particular with Model 3, since the filtered and smoothed probabilities also captures some periods of shocks, but the second chain does not represents the moments of discretionary monetary policy. This explains why the output gap series of this version was more alike to the result presented by Oliveira (2013).

5.1 Output Gap

Our approach takes us to five different output gap series, which are all represented in the Appendix B. The gap resulting from Model 0 does not capture very well the recession periods, except for the period 2003Q1-2003Q2. The series presents a volatile behavior, including positive output gap and peaks in notably recessive periods, as in the first and last ones considered by our sample. The output gap series from Models 1 and 3 also demonstrate the same pattern. By its turn, the gap of Model 4 was able to better capture the recessions of 2003 and 2008, in addition to presenting a much less volatile behavior than the others. The series was also the one that came closest to the result found by Oliveira (2013). However, it maintained the same pattern as the previous models in the 2001Q1-2001Q4 and 2014Q2-2016Q4 recessions, even showing an expansion trend in the former and peaks in the latter.

The best output gap series found by the models consider here was Model 2. In Figure 1 is possible to notice the difference between the output gap series under each regime, while the gray bars represent the recessive periods dated by CODACE. As an illustration, if the BCB followed a low inflation targeting monetary policy, the 2003 recession could have been more intense, in terms of the drop, and the 2008 financial crisis could have taken us to a lower level of output, as represented by Regime 2. But, as the filtered and smoothed probabilities[19] were unable to capture periods of low inflation targeting regime or discretionary monetary policy, as in 2003 and 2015, the output gap weighted by the smoothed probabilities was very similar with the Regime 1 series, as shown in Figure 2

[19] We opted to show all the probabilities in the appendix, since our focus is on the output gap series, and not on the period of occurrence of the regimes.

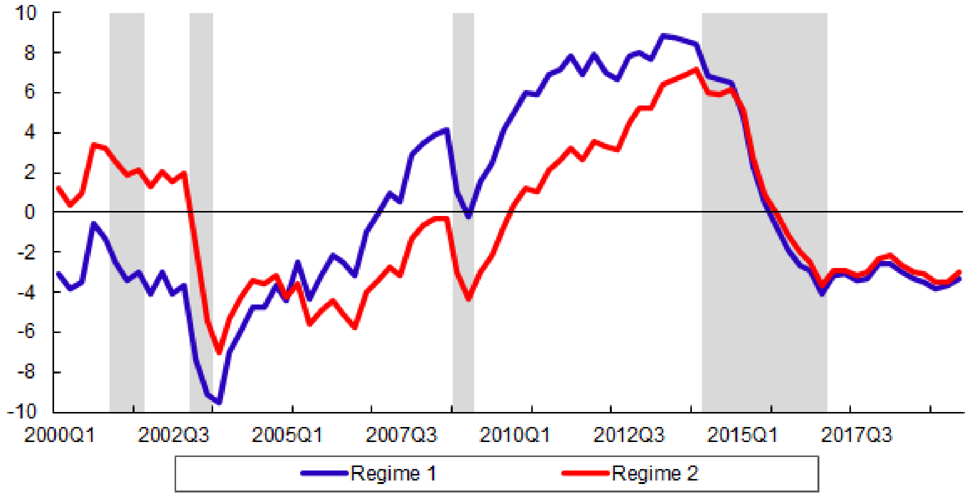

Figure 1 – Smoothed output gap series in each regime - Model 2

below. The Model 2 output gap series represents very well all the recessions periods dated by the CODACE and the series also demonstrates the size of the last recession, both in duration and in level, as discussed in the work of Pires, Borges e Borça Jr (2019).

In comparison with other works that also use the DSGE approach, like Justiniano e Primiceri (2008), Hirose, Naganuma et al. (2007) and Oliveira (2013), the latter two using the same model as this work, our output gap series proved to be less volatile than the others, especially when compared to Oliveira (2013) series, since both portray the brazilian economy, although our work uses a larger sample period. Oliveira (2013)'s results consider not only the recession periods dated by CODACE, but also the NBER definition of recession, as the period between the peak and the valley. The first difference that can be noticed in the comparison with this work is the 2003Q1-2003Q2 recession: while our results show that the output gap was already in negative territory in the period (like the other series presented here), Oliveira's results start from the positive to negative terrain, in a movement very similar to that observed in versions 1, 3 and 4 of our model. Our interpretation is that this result is highly contaminated by the inflationary shock of the period. In the second recessive period considered by the author, 2008Q4-2009Q1, the same movement occurs, starting from a positive peak to a negative one. Our results, on the other hand, also capture the recession, but to a much lesser extent: in the valley, our output gap is -0.2%, while the result presented by the author is around -1.3%. Finally, at the end of the sample used by the author, he points to a recessive period not considered by CODACE, which would go from 2010Q4-2012Q3 (end of his sample). However, our results do not capture this occurrence, being more adherent to the Committee.

Using the advantages of estimating the output gap through a DSGE approach, as demonstrated by Christiano, Eichenbaum e Trabandt (2018), we can interpret the series' behavior through the contribution of each shock. The same exercise was done by Oliveira (2013), in the output gap series, and Gonçalves, Portugal e Aragón (2016), in the output growth series. Figure 3 shows the output gap historical decomposition. Differently from what Oliveira (2013) concluded, this figure shows us that one of the main factors for the recession in the 2003Q1-2003Q2 period was the cost-push shock (σ_z), followed by the monetary shock (σ_r). In fact, in this period, the presidential dispute was taking place and the polls pointed to Luiz Inácio Lula da Silva, "Lula", as the next president of Brazil. This

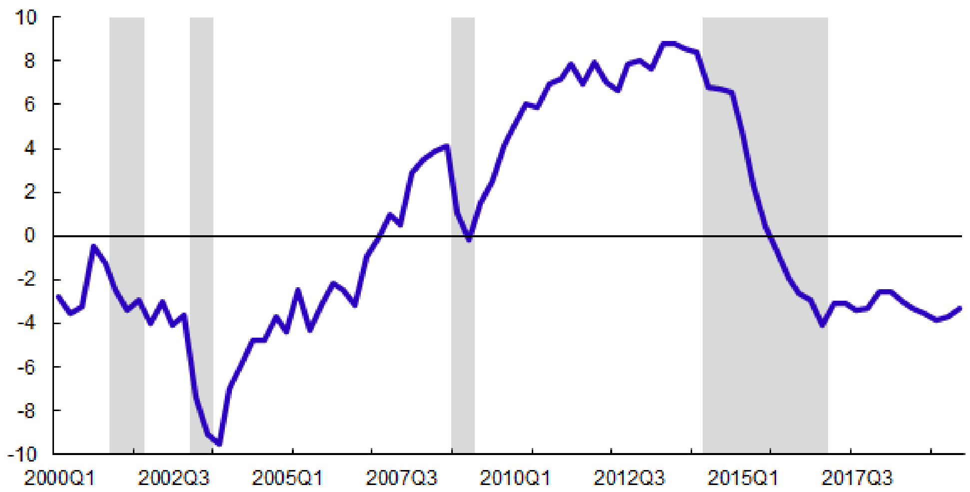

Figure 2 – Smoothed MS-DSGE output gap - Model 2

scenario "led to an episode of current account reversal, with a large devaluation of the real exchange rate and a sharp increase in the interest rates of government debt securities, in both the domestic and external debt markets" (AYRES et al., 2019). Such events occurred because Lula, in the past, defended the renegotiation of internal and external debts, which, in the eyes of the financial markets, sounded like the possibility of default. For these reasons, the exchange rate depreciated rapidly, passing such shocks on to the price level, which, in turn, explains the cost-push shock to the firms' price setting. Due to the scenario of uncertainty and inflationary shocks, the monetary authority began a cycle of monetary tightening, with impacts on the output gap being present until 2007Q3.

With the maintenance of macroeconomic stability during Lula's first term, the country was able to turn its growth trajectory, a movement favored by the worldwide boom in commodity prices. As we can see in Figure 3, between 2004 and 2008, Brazil had the best economic outcomes. However, the financial crisis of 2008 stopped this climb. The crisis triggered the recession of 2008Q4-2019Q1, which was strongly caused, according to our model, due to the great negative demand shock, since the moment was of pure uncertainty. After that, the country was still able to ride the commodities' wave, which can be seen in both positive cost-push and demand shocks. Since Brazil is a major exporter of commodities, the positive shock in prices was both a positive shock in costs and in demand, since the increase in commodity prices produced a positive wealth effect, due to the improvement in terms of trade. During this period, monetary shocks also contributed, especially between September 2011 and October 2012, in which the nominal interest rate went from 12.50 to 7.25.

However, here it is necessary to highlight how the government at that time faced the financial crisis. Lula was in his second term, but with a different macroeconomic policy than the first, with a strongly interventionist profile. And in its eagerness to shield the country from the damaging effects of the financial crisis, the government began to bet even more on these policies, on the idea that countercyclical policies could prevent the recession from being as damaging as it was showing for other countries, such as Portugal, Italy, Greece and Spain (PIGS). But all of these policies came with a price, and in 2012 the economy was already showing signs of exhaustion.

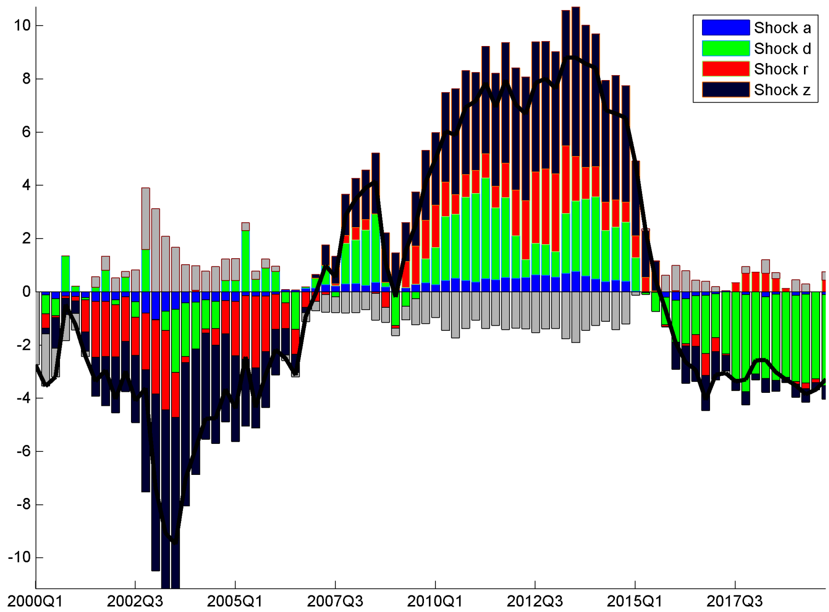

Figure 3 – Historical decomposition of the MS-DSGE output gap

Due to countercyclical policies, the fiscal situation deteriorated, and the use of *creative accounting* made the situation even worse, since the drop in commodity prices no longer allowed the adjustment of public accounts to be on the revenue side. The government, through the control of administered prices, such as fuels and electricity sold by SOEs, started to try to keep inflation artificially low, even with the free prices of the economy increasing. Intervention in SOEs occurred precisely because the government did not want to register rising inflation, also because the monetary authority was pressured by the government to reduce the nominal interest rate in this context. Further, by instructing public banks to pay social security pensions and by the incomplete reimbursement of the full amount of these payments, the public banks had losses that should, in fact, be counted as government's primary deficits. These fiscal maneuvers led to the impeachment of President Dilma Roussef in 2015 and the fiscal crisis of 2014Q2-2016Q4, from which the country has not yet recovered, as can be seen in Figure 2.

The historical decomposition of the period described above shows that, at first, the main force was the cost-push shock, as the uncertainty at the time caused both inflation and the nominal interest rate to increase dramatically: in December 2015, the variation in the price level registered 10.7% and the Selic, 14.25%. After that, the main force, which continues to prevail nowadays, was a negative and persistent shock of demand, since the economy has deteriorated so much that today we live with a high level of unemployment, higher rates of informality and a challenging scenario for the entrepreneur class.

The analysis is even more critical when we observe that at no point during the analyzed period, productivity played a relevant role in the trajectory of output gap. And, paradoxically, such a path seems to have been the only one left, because even if we consider the important sign of commitment to the fiscal adjustment embodied by the Spending

Ceiling[20] and the approval of the pension reform, the extremely rigid character of public accounts does not contribute for reversing this scenario.

5.1.1 Output Gap Analysis and Comparison

A good exercise is to compare the output gap found, which is based on the DSGE approach, with other available series, such as those from the IFI and IPEA, which are built using the production function approach, and the resulting output gap from an aggregate approach, such as the HP Filter.

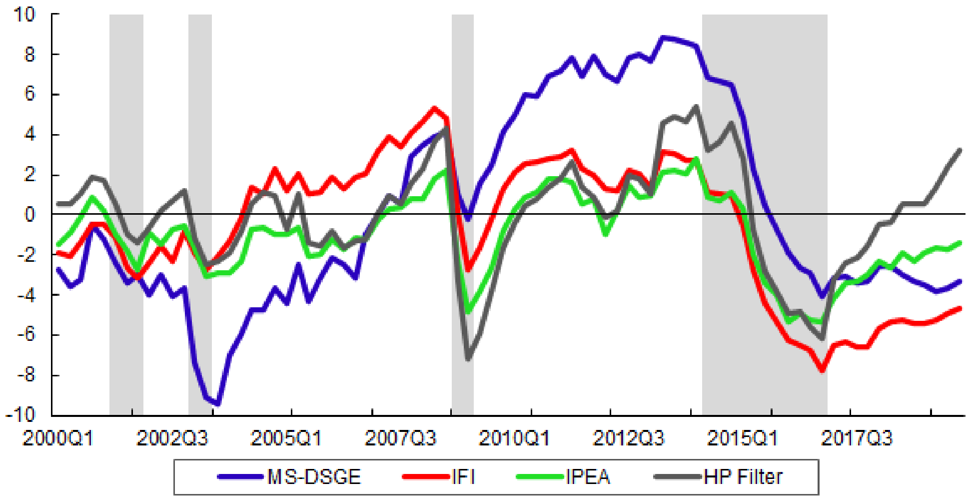

Figure 4 – Output gap series comparison

Figure 4 shows us that the series are similar to each other (also demonstrated in table 3), and that all are capable of capturing dated recessions. However, our series is detached from the others in terms of level. At the beginning of the sample, the MS-DSGE output gap is already more negative than the others and the 2003Q1-2003Q2 recession is more intense. This is because our inflation series[21] showed an outlier in the last quarter of 2002, due to the great uncertainty of the moment with the election results, which spilled over to expectations regarding the country's macroeconomic stance. Due to the abrupt fall, the activity recovery of the MS-DSGE series between 2003Q3-2008Q3 is more accelerated than that presented by the other series, but all reached a similar level before the financial crisis.

On the other hand, in the 2008Q4-2009Q1 recession, the MS-DSGE output gap series expresses a much less intense impact than the others. Such divergences may lie in the fact that our model represents a closed economy, so that the exchange rate movements of the period are not considered, which could help to better describe such recessions.

Finally, the 2014Q2-2016Q4 recession is captured in a very similar way by all series. Our MS-DSGE series and the HP Filter series show a steeper fall than the others, despite the IFI series reaching a lower level. Still, another interesting observation is that the aggregated approach series shows a quick recovery, as if the output gap was already positive again. Less intensely, the IPEA production function approach series shows a tendency to

[20] The constitutional amendment №95/2016 establishes a spending ceiling on the federal budget, whose growth is limited to the inflation of the previous year.

[21] See the Appendix.

close the gap, as does the IFI series, but at a much lower level. The MS-DSGE series, in turn, shows a much slower recovery than the others, with no clear sign of a reversal of the scenario.

Table 3 – Correlation Between Series

	Correlation			
	MS-DSGE	IFI	IPEA	HP Filter
MS-DSGE	1.00	0.5856	0.6938	0.5047
IFI	0.5856	1.00	0.8131	0.5658
IPEA	0.6938	0.8131	1.00	0.8893
HP Filter	0.5047	0.5658	0.8893	1.00

6 Prediction Tests

Although the graphical analysis of the series turns out to be a valid exercise, the best comparison is quantitative. In this sense, here it is proposed to carry out forecasting tests, similarly to Oliveira (2013). First, the central bank's reaction function will be used to verify which gap estimate, among those presented in this work, is more adherent to the interest rate actually observed. This exercise does not seek to show which output gap series is better, but rather try to identify which of the estimates is more consistent with the BCB's monetary policy decisions. In the next step, we will use the Phillips Curve to project the free items inflation, since the output gap can be a good measure of inflationary pressure, as Mishkin (2007) argues. The aim is to try to verify if the structural gap derived from the MS-DSGE model is a better predictor for future inflation, when compared to other approaches.

6.1 Central Bank's Reaction Funciton

The reaction function used is:

$$i_t = \beta_1 i_{t-1} + (1-\beta_1)(\beta_2 h_t + \beta_3(E_t\pi_t - \pi_t^*)) + \varepsilon_t \tag{44}$$

where i_t is the month effective Selic interest rate in annualized terms, h_t is the output gap, $E_t\pi_t$ is the expected inflation rate in t and π_t^* is the inflation target for t. However, taking into account the fact that in Brazil, the inflation target for t and $t+1$ is known by the BCB at the beginning of t, it is reasonable to assume[22] that monetary policy is guided by the inflation target for the current year and the subsequent. Thus, following Minella et al. (2003), we will use a weighted average of the expected inflation deviation from its target for years t and $t+1$, respectively, given by:

$$D_{jt} = \frac{4-j}{4}(E_j\pi_t - \pi^*) + \frac{j}{4}(E_j\pi_{t+1} - \pi_{t+1}^*) \tag{45}$$

[22] The monetary authority itself takes this stance in the monetary policy decision announcements of the Copom (Monetary Policy Committee).

where j is the quarterly index, $E_j\pi_t$ is the expected inflation for t in quarter j, $E_j\pi_{t+1}$ is the expected inflation for $t+1$ in quarter j, π^* is the inflation target for t e π^*_{t+1} is the inflation target for $t+1$. Therefore, the central bank's reaction function becomes:

$$i_t = \beta_1 i_{t-1} + (1-\beta_1)(\beta_2 h_t + \beta_3 D_{jt}) + \varepsilon_t \tag{46}$$

The sample for the estimation was constructed through three series:

- Effective Selic interest rate, annualized. Available at BCB[23].
- Inflation target, defined by the National Monetary Council (CMN) and available at the BCB[24].
- Expected inflation for IPCA, available at FOCUS Expectations System - BCB.

The data cover the period 2000Q2-2019Q4 and are in quarterly frequency. In order to assess the adherence of the estimates to the effective Selic rate, a reaction function was estimated for each output gap series, through the Ordinary Least Squares (OLS) methodology. We are aware that the most common strategy to deal with the problem of endogeneity is to estimate a reaction function (Taylor type) using the Generelized Method of Moments (GMM) method. However, as demonstrated by Stock e Yogo (2002), weak instruments lead to poor parameter identification and asymptotic results become a poor guide to the actual sampling distributions. Also, Carvalho, Nechio e Tristao (2019) argue in favor of OLS estimation for monetary policy rules. For the authors, the standard practice in the empirical literature of using lagged endogenous variables as instruments brings an additional complication when shocks are persistent, as is the case of monetary policy shock (see Tables 1 and 2), because instruments and shocks may be correlated, hampering the asymptotic properties of GMM estimates. Thereby, for each estimate, forecasts were made up to eight steps ahead and as evalution criterion, the root mean square error (RMSE) was adopted. In the table below, the regressions results are presented.

Table 4 – Central Bank Reaction Function - Taylor Rule - OLS

	Dependent Variable: $Selic_t$			
	MS-DSGE	IFI	IPEA	HP Filter
	(1)	(2)	(3)	(4)
Intercept	0.381 (0.476)	1.138^{***} (0.367)	0.701^{*} (0.369)	0.407 (0.407)
$Selic_{t-1}$	0.914^{***} (0.042)	0.847^{***} (0.034)	0.906^{***} (0.035)	0.917^{***} (0.038)
Djt	0.416^{***} (0.126)	0.559^{***} (0.119)	0.443^{***} (0.117)	0.397^{***} (0.121)
Gap_t	0.054^{*} (0.029)	0.117^{***} (0.032)	0.200^{***} (0.057)	0.137^{***} (0.045)
Observations	68	68	68	68
R^2	0.944	0.951	0.951	0.948
Adjusted R^2	0.941	0.949	0.948	0.946
Residual Std. Error (df = 64)	0.988	0.920	0.929	0.949
F Statistic (df = 3; 64)	360.233^{***}	418.387^{***}	410.790^{***}	392.252^{***}
Note:	$^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$			

As it can be seen in Table 5, the reaction function which used the HP Filter output gap series presented the lowest RMSE among the others, until six periods ahead, followed by the reaction function with the MS-DSGE series. But, for long-term forecasts, the DSGE approach showed better results from seven steps ahead. The output gap derived from the

[23] Series code 4189.
[24] Series code 13521.

production function approaches based on the Orair e Bacciotti (2018) and Souza-Júnior (2017) works presented low predictive power in relation to the others.

It is worth noting that even though the MS-DSGE series does not have the lowest RMSE for short-term forecasts, the adjustment was significantly better than that of the production function approach. Still, it is necessary to remember that for the purposes of monetary policy, it is extremely important to understand the forces that act on the variables used to guide monetary policy decisions, and in this respect we cannot count on the HP filter series.

Table 5 – Central Bank Reaction Function - RMSE

	Root Mean Square Error - RMSE							
	$h = 1$	$h = 2$	$h = 3$	$h = 4$	$h = 5$	$h = 6$	$h = 7$	$h = 8$
MS-DSGE	0.1666	0.1488	0.2116	0.2389	0.2841	0.3309	0.3229	0.4201
IFI	0.4645	0.3836	0.3912	0.3976	0.4315	0.4671	0.4408	0.5107
IPEA	0.9094	0.8496	0.8732	0.8919	0.9248	0.9526	0.8866	0.8385
HP Filter	0.0273	0.0199	0.0186	0.0162	0.0483	0.0489	0.3322	0.6066

6.2 Phillips Curve

The output gap estimates were used to forecast the free items inflation using the Phillips Curve, in the same proposal of Oliveira (2013)[25]. The aim is to verify if the structural output gap is a better predictor for inflation, since the output gap can serve as a measure of inflationary pressure. For this, the Phillips Curve used was:

$$\pi_t^L = \beta_1 \pi_{t-1} + \beta_2 E_t \pi_{t+1} + \beta_3 h_{t-1} + \varepsilon_t \tag{47}$$

where π_t^L is the inflation of free items, π_t is the general inflation rate, $E_t\pi_{t+1}$ is the expectation in t of the general inflation for $t+1$ and h_t is the output gap.

The sample data to perform the estimation were:

- Free IPCA inflation, annualized, available at BCB[26].
- General IPCA inflation, annualized, available at IBGE.
- Smoothed expected IPCA inflation series, for $t+1$, available at FOCUS Expectations System - BCB.

The data cover the period 2001Q4-2019Q4 and are in quarterly frequency. The free items IPCA inflation was chosen because this index is more sensitive to monetary policy, compared to the general index. Here, the estimation was performed through GMM method and the choice of instruments was based on the proposal of Mendonça, Sachsida e Medrano (2012). Thus, the set of instruments used consists of lags up to the third order of general inflation, unemployment and nominal interest rate (Selic rate). In the same way as

[25] It uses the mean square error (MSE) as an evaluation criterion.
[26] Series code 11428.

was performed in the reaction function exercise, for each output gap series, forecasts were made up to eight steps ahead and we also used the RMSE as the evalution criterion.

Similarly to Oliveira (2013), no output gap series stands out among the group, so that a separate analysis by forecast horizon is justified. For one and two steps ahead, the HP Filter output gap presented a best perform, especially at two steps ahead. But at a forecast three and four steps ahead, the MS-DSGE was the series that added more information to the forecast. In its turn, the IFI output gap presented a better perform for six and seven steps ahead. By last, the HP Filter series stands out in the last forecast window. It is necessary to remember that these results are only comparative, since we are analyzing estimates that were not made from the same sample.

Table 6 – Phillips Curve - GMM

	Dependent Variable: $Free_t$			
	MS-DSGE	IFI	IPEA	HP Filter
	(1)	(2)	(3)	(4)
Intercept	-2.840^{***} (0.728)	-1.515^{**} (0.607)	-0.645 (0.847)	-1.767^{***} (0.643)
$Free_{t-1}$	0.455^{***} (0.071)	0.516^{***} (0.062)	0.667^{***} (0.043)	0.697^{***} (0.053)
$Exp.Smooth_t$	1.106^{***} (0.131)	0.873^{***} (0.115)	0.554^{***} (0.145)	0.622^{***} (0.132)
Gap_{t-1}	0.137^{***} (0.028)	0.154^{**} (0.063)	0.367^{***} (0.096)	0.330^{***} (0.102)
J-Test	5.08	8.36	4.06	2.47
J-Test (p-valor)	0.53	0.21	0.67	0.87
Observations	69	69	69	69
Note:	*p<0.1; **p<0.05; ***p<0.01			

Table 7 – Phillips Curve - RMSE

	Root Mean Square Error - RMSE							
	$h = 1$	$h = 2$	$h = 3$	$h = 4$	$h = 5$	$h = 6$	$h = 7$	$h = 8$
MS-DSGE	1.1139	0.9401	0.8282	0.7183	0.8404	0.7992	0.8171	0.9776
IFI	1.1426	0.9466	0.8851	0.7913	0.8284	0.7581	0.7241	0.8284
IPEA	1.1262	0.9123	0.9018	0.8864	0.8687	0.8122	0.7250	0.8070
HP Filter	0.9246	0.7739	0.8547	0.8909	0.8272	0.7889	0.7380	0.7111

7 Conclusions

The objective of this work was to contribute with the literature of output gap estimation, an unobservable variable that, due to this characteristic, presents different methodologies for its estimation. In this work, we estimated the output gap based on a fully specified DSGE model that incorporates Markov-Switching elements. This model-based estimation is a good measure for welfare since it its derived from optimizing behaviour of the agents. Also, the approach permits the estimation of structural parameters and the access to fundamental shocks, which allow an economic interpretation for movements of the estimated output gap.

In particular, we proposed four versions of the model and the one that best captured the recession periods that Brazil went through between 2000Q1-2019Q4 was that in which Taylor rule parameters were allowed to change, for one regime of high inflation targeting to another of low inflation targeting. In the historical decomposition analysis, we find that cost-push and demand shocks were the main forces in the output gap path for the period considered. Also, when comparing our MS-DSGE estimate with public production function approaches and the standard HP Filter estimate, we noticed that the MS-DSGE output gap presents some advantage for long-term forecasts, although the aggregate approach example perform better on shorter horizons.

No doubt, this work has its limitations. Despite the use of an MS-DSGE approach being a novelty to estimate the output gap in the brazilian case, the model considered here does not portray a small open economy and also does not make use of any fiscal policy variable, which could add to the estimate. Also, we set some of our priors based on constant parameter DSGE models, which could not be the same in a switching approach. The adoption of exogenous transition probabilities is also a deficiency. We are aware that it would be better to estimate all the parameters together, including the probabilities, so that the optimizing behaviour also extended to the behavior of the monetary authority, which would choose what regime to follow for each state of the economy. We hope that, in a future work, we can overcome such limitations.

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APPENDIX A – Data

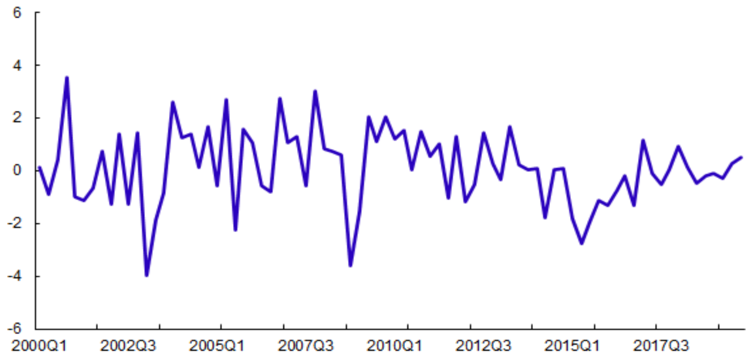

Figure 5 – Output per capita Growth

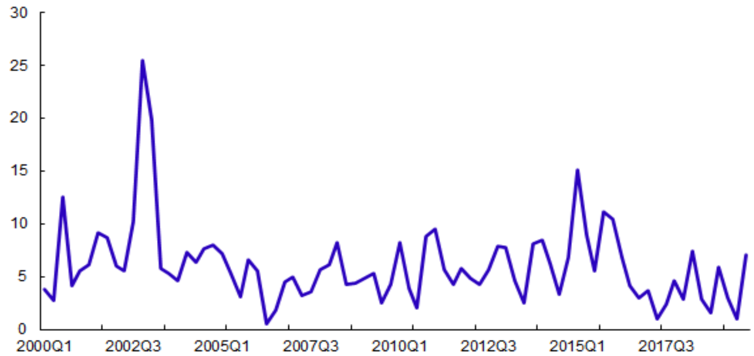

Figure 6 – Quarterly Inflation Rate - IPCA

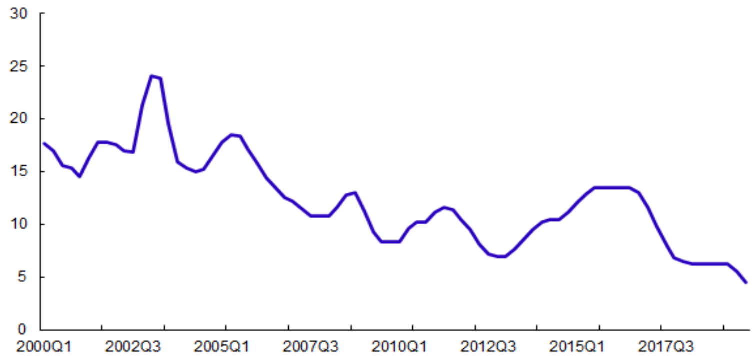

Figure 7 – Quarterly Nominal Interest Rate - Selic

APPENDIX B – Smoothed Gaps and Probabilities

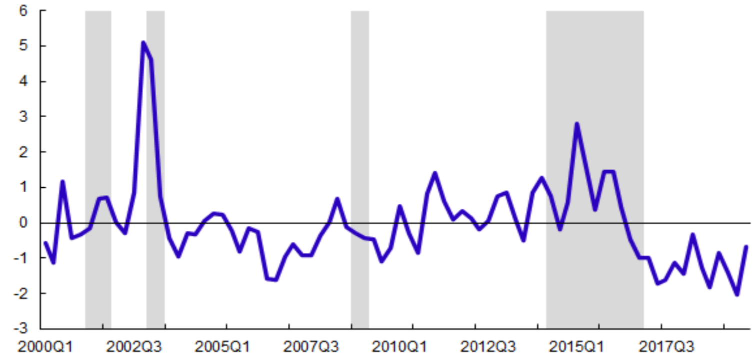

Figure 8 – Model version: DSGE model with no regime switching

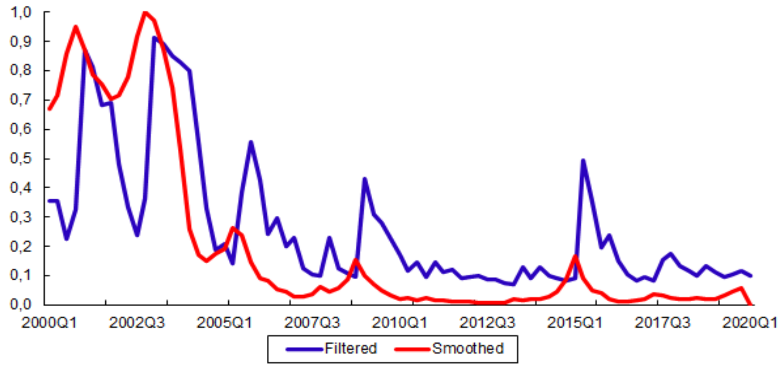

Figure 9 – Model version: Regime switching in volatilities only - Probabilities Regime 2

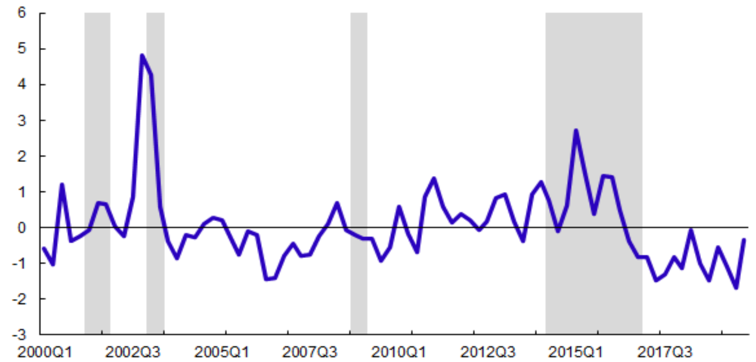

Figure 10 – Model version: Regime switching in volatilities only - Smoothed Output Gap

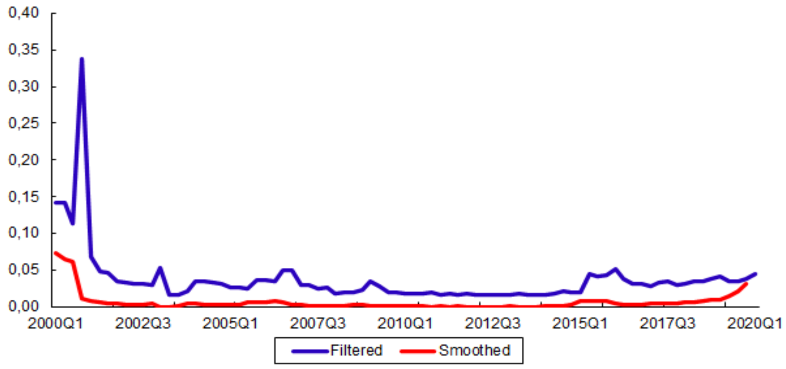

Figure 11 – Model version: Regime switching in Taylor rule parameters only - Probabilities Regime 2

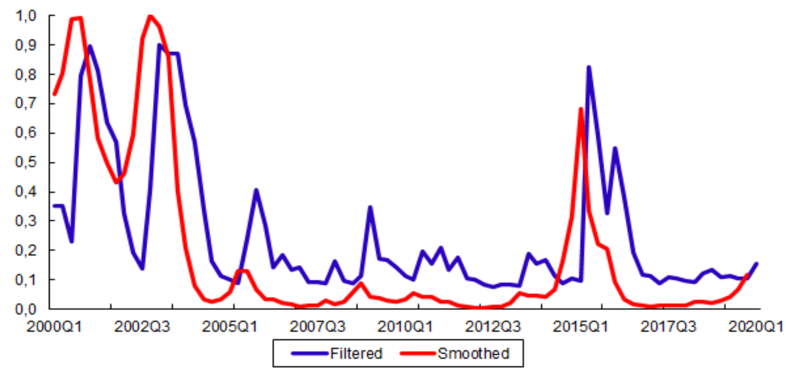

Figure 12 – Model version: Regime switching in Taylor rule and volatilities - Probabilities Regime 2

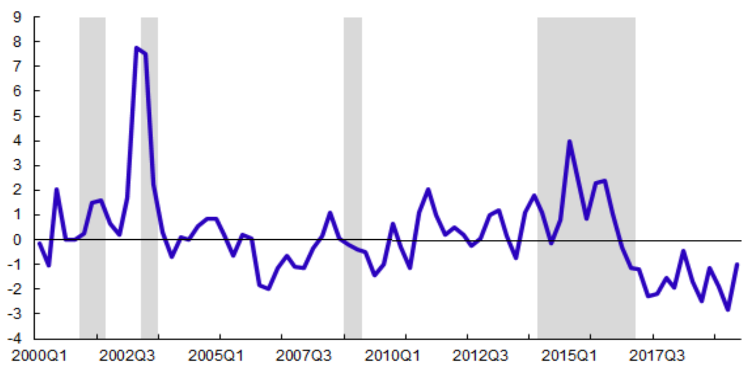

Figure 13 – Model version: Regime switching in Taylor rule and volatilities - Smoothed Output Gap

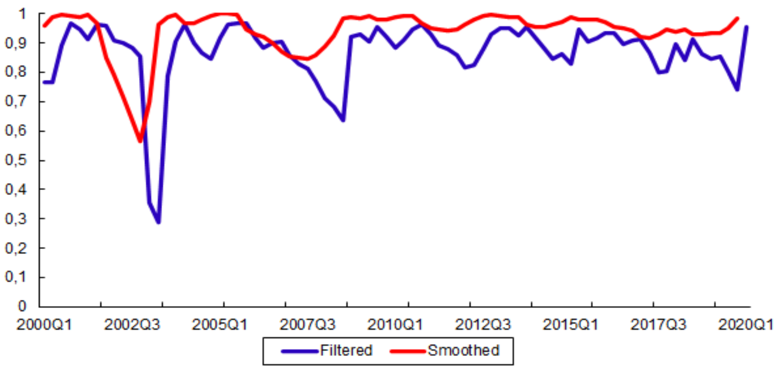

Figure 14 – Model version: Regime switching in Taylor rule and volatilities with Independent Chains - Probabilities Regime 2 of the first chain

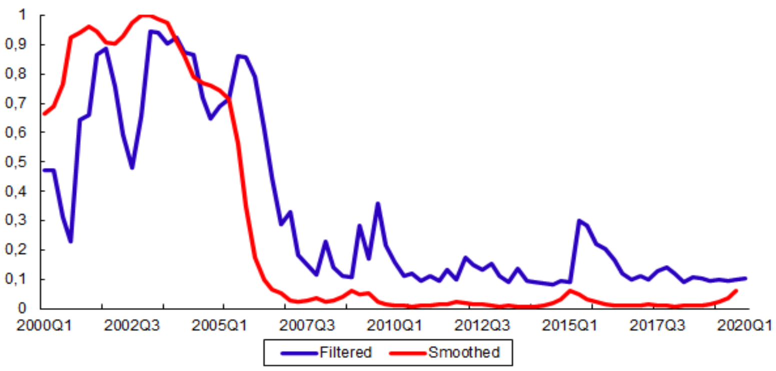

Figure 15 – Model version: Regime switching in Taylor rule and volatilities with Independent Chains - Probabilities Regime 2 of the second chain

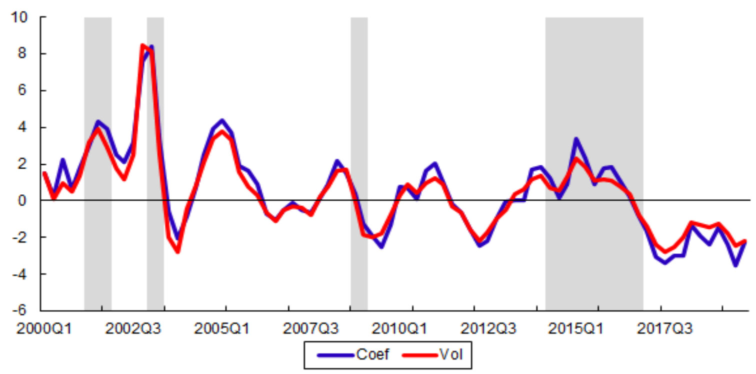

Figure 16 – Model version: Regime switching in Taylor rule and volatilities with Independent Chains -Smoothed Output Gap